# NOTES AND EXAMPLES ON THE THEORY OF HEAT & HEAT ENGINES

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# PREFACE.

THE present volume is an enlargement of my "Synopsis of the Elementary Theory of Heat and Heat Engines," which has been out of print for some time. The most important new feature is the addition of a large number of examples worked out in the text, with many others for the student to work out himself. I hope that these additions will considerably increase the value of the book.

The book is not intended as an exhaustive text-book, as sufficient excellent treatises exist already, but rather as a companion to lectures, to help the student to see at a glance the important points of the subject, and to assist him with his revision for examinations. I hope also that the practical engineer, who has to deal with the elementary thermodynamics of heat engines will find the book of value, and to this end all the more important formulae are printed in heavy type.

I am greatly indebted to Mr. C. Barclay-Smith for his labour of proof reading.

JOHN CASE.

BRISTOL, 1922.

# CONTENTS.

## GENERAL PRINCIPLES I.

1. If heat be supplied to a system, the system may perform work; if work be done on a system, heat may be produced. Therefore heat is a form of energy.

2. The effects of supplying a system with heat are in general (i.) to change the physical state* of the system; (ii.) to make the system do work.

The energy of a system depends only on the physical state of the system.

Changing the physical state of a system changes its store of energy. In general we are here only concerned with its store of thermal energy. All the energy which is not kinetic is called internal energy.

3. The conservation of energy gives:

| | | | | | | |
|---|---|---|---|---|---|---|
| Total heat energy absorbed by a system | = | increase of kinetic energy, if any | + | work done by the system | + | increase of internal energy |

whenever heat is supplied to a system.

Hence (i) if there be no increase of kinetic energy and no work done we can define **internal energy** by saying:

4. When a system absorbs heat without doing external work or increasing its kinetic energy, it is said to change its internal energy by an amount equal to the heat energy absorbed.

(ii.) If there be no change of kinetic or internal energy, all the heat energy absorbed is converted into work energy. Thus we arrive at

5. **The First Law of Thermodynamics:** When work is performed by a system at the expense of heat energy, for every unit of heat that goes out of existence one

* The term 'physical state' is meant to include such things as position, velocity, etc., besides the thermal, electrical, etc. state.

unit of work is performed, and conversely, provided of course they are both measured in the same units.

But we generally measure heat energy on one scale and mechanical energy on another, and a correcting factor is necessary; this we call **J.**

6. **The Unit of Heat** that we shall employ is the quantity of heat necessary to raise the temperature of one pound of water 1° C. This is called a Standard Thermal Unit, and is equal to 1400 ft.-lbs. Hence on these scales

$$\mathbf{J = 1400.}$$

1 Th. Unit = 1400 ft.-lbs.

1 Ft.-lb. = 1/1400 Th. Unit.*

Sometimes this unit of heat is called a pound-calorie.

7. **The Specific Heat** of a substance is the ratio of the heat required to raise a mass of the substance through a certain temperature rise to that required to raise by the same amount the temperature of an equal mass of water.

By §6 the Sp. Ht. is numerically equal to heat required to raise the temperature of 1lb. of the substance 1° C.

8. Let $dQ$ = heat absorbed by a system from without in any change.

" $dW$ = the work done by the system.

" $dE$ = the increase of internal energy.

Let there be no change of kinetic energy.

Then, by § 3:

$$\mathbf{dQ = dW + dE.}$$

9. An **adiabatic change** is one in which no heat is absorbed or given up, and then

$$dW + dE = 0.$$

An **isothermal change** is a change at constant temperature.

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* Sometimes A is used for 1/J.

10. A **cycle** is a series of changes such that at the end of the series the state of the substance undergoing the change is the same as before the changes commenced.

11. **Absolute temperature.** The absolute zero is the temperature at which a perfect gas would have zero volume. It is 273° below the freezing point of water on the centigrade scale. Hence, if $T$ = the absolute temperature, and $t$ the centigrade temperature,

$$\mathbf{T = t + 273.}$$

## EXAMPLES.

**1.** If water is forced through a porous plug under a pressure of 1000 lbs./in.$^2$, find the approximate rise of temperature on the supposition that all the heat produced remains in the water (1 cubic foot of water weighs $62\frac{1}{2}$ lbs.) (Special Exam. Cambridge, 1910.)

Suppose the area of the section of the porous plug is one square foot, and that the velocity with which the water passes through is $v$ ft./sec.

Then the work done $= 1000 \times 144\, v$ ft. lbs./sec.

$$= \frac{144{,}000}{1400}\, v = 103\, v \text{ Th. Units/sec.}$$

The volume of water dealt with $= v$ ft.$^3$/sec.

$\therefore$ ,, weight ,, ,, ,, ,, $= 62{\cdot}5\, v$ lbs./sec.

Let $t$°C. be the rise in temperature, then

$$62{\cdot}5\, v \times 1 \times t = 103\, v.$$

$$\therefore \qquad t = \frac{103}{62{\cdot}5} = 1{\cdot}65.$$

**2.** In turning a steel shaft, 4 lbs. of metal are removed per minute. The cutting speed is 200 ft. per minute, and the pressure on the point of the tool in the direction of cut is 1700 lbs. Assuming that all the work done by the tool goes to heating the turnings, find their increase of temperature as they leave the tool. The specific heat of steel may be taken as 0·12. (Special Exam. Cambridge, 1912.)

The work done per minute $= 1700 \times 200 = 340{,}000$ ft. lbs.

$$= \frac{340{,}000}{1400} = 243 \text{ Th. Units.}$$

Let $t$°C. = the rise of temperature of the turnings. Then

$4 \times 0{\cdot}12 \times t =$ the heat given to the turnings

$= 243$ Th. Units.

$$\therefore \qquad t = \frac{243}{0{\cdot}48} = 500\text{°C. approximately.}$$

**3.** 1000 gallons of water are pumped per minute to a height of 80 ft. The pumps have an efficiency of 60%, and are driven by a steam engine. Find the H.P. of the engine. The kinetic energy of the water may be neglected. If the engine and boiler use 18,000 Th. Units per H.P. per hour, find the over-all efficiency of the whole plant. A gallon of water weighs 10 lbs.

The work done by the pumps

$$= 10{,}000 \times 80$$
$$= 800{,}000 \text{ ft. lbs. per minute.}$$

$\therefore$ The work done by the engine

$$= \frac{800{,}000 \times 100}{60}$$
$$= 1{,}333{,}000 \text{ ft. lbs. per minute.}$$
$$= 40 \cdot 6 \text{ H.P.}$$

The heat supplied to the boiler

$$= \frac{40 \cdot 6 \times 18{,}000}{60}$$
$$= 12{,}200 \text{ Th. Units per minute.}$$

In work units this is

$$12{,}200 \times 1400 = 17{,}080{,}000 \text{ ft. lbs. per minute.}$$

$\therefore$ the over-all efficiency

$$= \frac{\text{work done}}{\text{work supplied}} = \frac{800{,}000}{17{,}080{,}000}$$
$$= \cdot 0468$$
$$= 4 \cdot 7\ \% \text{ approx.}$$

**4.** An iron bullet is fired obliquely at a hard steel plate with a velocity of 1600 ft./sec. The bullet is deformed but not broken, and its velocity after striking is 800 ft./sec. The plate is unaffected. If the temperature of the bullet before striking is 20°C., shew that its temperature after striking is about 190°C. The average sp. ht. of iron is 0·12 for the range 20° to 100°, and 0·13 for the range 100° to 200°C. (Mech. Sci. Trip. Cambridge, 1913.)

The heat acquired by the bullet is equal to its loss of kinetic energy, which is

$$\frac{W}{64 \cdot 4}(1600^2 - 800^2) = 29{,}800\ W \text{ ft. lbs.}$$

where $W$ is the weight of the bullet.

In heating the bullet from $20^\circ$ to $100^\circ$ the heat absorbed

$$= W \times 0 \cdot 12 \times 80 = 9 \cdot 6\ W \text{ Th. Units.}$$

In heating it from $100^\circ$ to $t^\circ$C., the heat absorbed is

$$W \times 0 \cdot 13 \times (t - 100) = W\,(0 \cdot 13t - 13).$$

Hence $W(0 \cdot 13t - 13) + 9 \cdot 6\ W = \dfrac{29800}{1400}\ W = 21 \cdot 3$

$$\therefore \qquad 0 \cdot 13t = 21 \cdot 3 + 13 - 9 \cdot 6 = 24 \cdot 7$$

$$\therefore \qquad t = 190^\circ\text{C.}$$

**5.** An engine develops 500 H.P., the power being absorbed by a brake. The brake is cooled by a stream of water which enters the brake drum at 20°C., and leaves it at 80°C. How much water is used per minute? (Special Exam. Cambridge, 1912.)

**6.** A shell has an average specific heat of 0·1. It hits a target with a velocity of 2000 ft./sec., and is brought to rest by it. If the heat is at the moment all in the shell, find the rise of temperature that ensues. (Special Exam. Cambridge, 1913.)

**7.** Water, flowing in a pipe, with velocity $v_1$ ft./sec., passes abruptly into a pipe of larger section where its velocity is $v_2$ ft./sec. The loss of head due to the sudden enlargement is $(v_1 - v_2)^2/2g$ feet. If all the energy dissipated is expended in heating the water, find the rise of temperature when 40 cubic feet of water pass per second from a pipe of sectional area 1 sq. ft. to one of area 4 sq. ft. (Intercoll. Exam. Cambridge, 1908.)

**8.** An iron wire is suddenly loaded to a stress of 40,000 lbs./in.$^2$, and stretches under this load by $\frac{1}{20}$ of its length. Shew that the temperature rises about 4°C. The sp. ht. of the wire is 0·11, and its density 480 lbs. per cubic foot. (Mech. Sc. Trip., 1914.)

## PERFECT GASES. I.

12. The Characteristic Equation of a perfect gas is

$$\mathbf{pv = RT = R(273 + t)}$$

where
$$\begin{cases} p = \text{the pressure in lbs. per ft.}^2 \\ v = \text{volume of 1 lb. in cubic ft.} \\ t = \text{temperature °C.} \\ T = \text{absolute temperature.} \\ R = \text{a constant.} \end{cases}$$

In calculations we must take

$$144pv = R(273 + t)$$

when $p$ is in lbs. per sq. inch.

For air at 0° C, 760 mm., $R = 96$, $v = 12{\cdot}4$, using lbs. and feet as units.

13. The *Specific Heat* of a gas has different values according as heat is absorbed at constant pressure or constant volume.

$k_p$ = sp. ht. at constant pressure

$k_v$ = sp. ht. at constant volume.

For air $k_p = {\cdot}237$, $k_v = {\cdot}169$, both in heat units.

14. The **Internal Energy of a Gas:—**

(i.) *Joule's Experiment.* Joule took two vessels and filled one with air under pressure, while the other had a vacuum, the two being connected by a pipe with a tap in it. The tap was opened and the air expanded into the empty vessel; no change of temperature occurred, after the air had come to rest.

Now (a) no work was done; no heat was absorbed or given out; there was no final change of kinetic energy.

∴, by the energy equation (§3), the internal energy had not changed.

But (b) the pressure and volume *had* changed, the temperature did *not* change, and the internal energy had *not* changed.

$\therefore$ Conclusion: the internal energy of a gas depends only on the temperature.

(ii.) Take the case of gas absorbing heat at constant volume and so not doing any work.

By §7 we have

change of internal energy

= heat taken in by gas

= $k_v \times$ mass $\times$ change of temperature.

$\therefore$ for one lb. of gas:

increase of internal energy is given by

$$\mathbf{E_2 - E_1 = K_v (T_2 - T_1)} \text{..............Th.U.}$$

This is true in whatever manner the gas takes in heat.

15. **Work done by gas in Expanding at constant pressure.** Suppose we have a pound of gas shut up in a cylinder behind a piston which fits the cylinder closely.

Let $S$ = section of cylinder (sq. ft.),

$l_1$ = length of cylinder occupied by gas before expansion (ft.),

$l_2$ = ditto after expansion (ft.),

$p$ = pressure of gas (lbs./ft$^2$.)

Supply heat to gas in such a way that $p$ is constant.

Total force on piston $= pS$,

distance moved by piston $= l - l_1$,

$\therefore$ work done $= pS(l_2 - l_1)$

$= p(Sl_2 - Sl_1)$

$= p(v_2 - v_1) = R(T_2 - T_1)$ Ft.-lbs.

where $v_2$ and $v_1$ are the volumes of the gas before and after expansion. In a small change, when $v_1 = v$, and $v_2 = v + dv$, the work done is $pdv$, or, expressed in thermal units, $pdv/J$.

16. **To prove that $k_p - k_v = R$,**
let one pound of gas expand, behind a piston, at constant pressure. Then

$$Q = \text{heat received} = k_p\,(T_2 - T_1). \quad \text{Th. Units,}$$
$$W = \text{work done} = R\,(T_2 - T_1) \ \text{Ft. lbs.} \quad |\text{by §15.}$$
$$= \frac{R}{J}(T_2 - T_1) \quad \text{Th. Units;}$$

$\therefore$, by §8,

$$E = \text{change of internal energy,}$$
$$= Q - W = k_p(T_2 - T_1) - \frac{R}{J}(T_2 - T_1),$$
$$= \left(k_p - \frac{R}{J}\right)(T_2 - T_1);$$

also $$E = k_v\,(T_2 - T_1) \text{ by §14,}$$
$$\therefore \quad k_p - \frac{R}{J} = k_v,$$
or $$k_p - k_v = R,$$
if $R$ be supposed already expressed in Th. Units.

17. **Adiabatic Equation for a gas: to prove $pv^\gamma$ = constant,**

where $$\gamma = \frac{k_p}{k_v}.$$

Take a small change in which

$$\begin{cases} p \text{ remains constant,} \\ v \text{ changes to } v + dv, \\ T \quad ,, \quad ,, \ T + dT; \end{cases}$$

then $$\begin{cases} dQ = 0 \text{ since change is adiabatic,} \\ dW = pdv \text{ (ft.-lbs.) [by §17 (i.)]} = \dfrac{pdv}{J} \quad \text{Th. Units,} \\ dE = k_v dT \text{ (Th. U.) (by §14);} \end{cases}$$

$$\therefore \quad \frac{pdv}{J} + k_v dT = 0 \text{......(by §8)} \quad \text{.........(i.)}$$

but
$$pv = RT,$$

$$\therefore\ p = \frac{RT}{v} \quad \text{........................ (ii.)}$$

and $\log p + \log v = \log T + \text{const.}$ ..........................(iii.)

From (i.) and (ii.) :—

$$\frac{RT}{J}\frac{dv}{v} + k_v dT = 0,$$

$$\therefore\ \frac{R}{J} \cdot \frac{dv}{v} + k_v \frac{dT}{T} = 0$$

$$\therefore\ \frac{R}{J} \log v + k_v \log T = \text{constant};$$

or
$$(k_p - k_v) \log v + k_v \log T = \text{constant}.$$

$$\therefore\ (k_p - k_v) \log v + k_v (\log p + \log v) = \text{constant, by (iii.)}$$

$$\therefore\ k_p \log v + k_v \log p = \text{constant},$$

$$\therefore\ \frac{k_p}{k_v} \log v + \log p = \text{constant},$$

$$\therefore\ pv^{\frac{k_p}{k_v}} = \text{constant},$$

or
$$pv^{\gamma} = \text{constant},$$

where
$$\gamma = \frac{k_p}{k_v} = 1{\cdot}406 \text{ for air.}$$

## 18. **Isothermal Expansion of a Gas.** ($T$ = constant).

(i.) *Work done.* Suppose that, while the volume undergoes a small change from $v$ to $v + dv$, the pressure remains constant.

We have $pv = RT$

$= \text{constant since } T \text{ is constant.}$

B

Work done in small change $= pdv$.

$\therefore$ „ „ „ whole „ from $(p_1v_1)$ to $(p_2v_2)$.

$$= W = \int_1^2 p\,dv,$$

$$= \int_{v_1}^{v_2} RT\frac{dv}{v},$$

$$= RT \log_e \frac{v_2}{v_1},$$

*i.e.* $$\mathbf{W = p_1v_1 \log_e \frac{v_2}{v_1} = RT_1 \log_e \frac{v_2}{v_1}}$$

(ii.) *Change of Internal Energy.* $T$ is constant, $\therefore$ there is no change of internal energy.

(iii.) *Heat supplied* = work done + change of E.

$$Q = RT_1 \log_e \frac{v_2}{v_1}.$$

In isothermal compression $W$ is the work done on the gas, and $Q$ is the heat given out.

19. **Expansion of a gas in general.** The general law of expansion is *

$$pv^n = C,$$

where $n$ and $C$ are constants.

(i.) *Work done* $$= \int_1^2 p\,dv = \int_{v_1}^{v_2} C\frac{dv}{v^n}$$

or $$W = \frac{C(v_2^{1-n} - v_1^{1-n})}{1-n},$$

Now $$C = p_1v_1^n = p_2v_2^n.$$

$$\therefore \mathbf{W = \frac{p_2v_2 - p_1v_1}{1-n} = \frac{p_1v_1 - p_2v_2}{n-1}}$$

$$\mathbf{= \frac{R(T_1 - T_2)}{n-1}.}$$

* Not including the case of unresisted expansion.

(ii.) *Change of Temperature.*

$$p_2 v_2^{\,n} = p_1 v_1^{\,n}, \qquad \text{(i.)}$$

and

$$\frac{p_2 v_2}{T_2} = R = \frac{p_1 v_1}{T_1}, \qquad \text{(ii.)}$$

$$\therefore\ v_2^{\,n-1} T_2 = v_1^{\,n-1} T_1 \qquad \text{by division of (i.) by (ii.)}$$

$$\therefore\ \mathbf{T_2} = \left(\frac{\mathbf{v_1}}{\mathbf{v_2}}\right)^{\mathbf{n-1}} \mathbf{T_1} = \left(\frac{\mathbf{p_2}}{\mathbf{p_1}}\right)^{\frac{\mathbf{n-1}}{\mathbf{n}}} \mathbf{T_1}.$$

(iii.) *Change of Internal Energy.*

$$E = k_v(T_2 - T_1).$$

(iv.) *Heat Supplied* $= W + E,$

$$= \frac{R(T_1 - T_2)}{n - 1} + k_v(T_2 - T_1),$$

$$= \left(\frac{R}{n-1} - k_v\right)(T_1 - T_2).$$

20. **Adiabatic Expansion of a Gas.** Put $n = \gamma$ in above:

$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = \frac{R(T_1 - T_2)}{\gamma - 1},$$

$$Q = 0.$$

$$\therefore\ E = -W,$$

$$T_2 = \left(\frac{v_1}{v_2}\right)^{\gamma-1} T_1 = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} T_1.$$

## EXAMPLES.

**1.** One pound of gas at 15°C., 20 lbs. per sq. inch absolute pressure is enclosed in a cylinder with a moveable piston. The gas occupies 9·65 ft.³ 24 Th. Units of heat are supplied to the gas, and the temperature rises to 92°C., the pressure being kept constant. Find the external work done, and the increase of internal energy. Find also the specific heat at constant volume.

The new volume $= \dfrac{273+92}{273+15} \times 9{\cdot}65 = 12{\cdot}3$ ft.³

$p \times 144 \,( V_2 - V_1)$

The work done $= 20 \times 144\,(12{\cdot}3 - 9{\cdot}65)$

$= 7660$ ft. lbs.

$= 5{\cdot}47$ Th. Units.

Increase of internal energy = heat supplied − work done.

$= 24 - 5{\cdot}47$

$= 18{\cdot}53$ Th. Units.

This also $= k_v\,(92 - 15) = 77k_v$.

$\therefore \qquad k_v = \dfrac{18{\cdot}53}{77} = 0{\cdot}241.$ $\dfrac{\Delta E}{T_2 - T_1}$

**2.** For a certain gas, which may be assumed perfect, the weight of a cubic foot, at 0°C., 14·7 lbs. per sq. in. pressure, is found to be 0·0893 lbs., and the sp. ht. at constant volume is 0·155. Find the value of $\gamma$. (Intercoll. Exam. Cambridge, 1911.)

We have

$v$ = the volume of one pound $= \dfrac{1}{0{\cdot}0893} = 11{\cdot}2$ ft.³

$p = 14{\cdot}7 \times 144$ lb. ft.²

$T = 273.$

From §12, we have

$$R = \frac{pv}{T} = \frac{14{\cdot}7 \times 144 \times 11{\cdot}2}{273} = 86{\cdot}8 \text{ ft. lb. units.}$$

$$= 0{\cdot}062 \text{ Th. Units.}$$

From §16, $k_p = R + k_v = 0{\cdot}217$

$$\therefore \qquad \gamma = \frac{k_p}{k_v} = \frac{0{\cdot}217}{0{\cdot}155} = 1{\cdot}40.$$

**3.** A cylinder, provided with a piston, contains 1 lb. of dry air at 273°C. and 367·5 lbs./in.$^2$ absolute. The air expands adiabatically to five times its original volume; it is then compressed isothermally to its original volume, and the cycle completed by supplying heat at constant volume until the initial conditions are obtained. Find (1) the work done in foot-pounds, (2) the heat taken in and rejected. (Mech. Sc. Trip., 1910.)

We shall use the equations of § 20 for dealing with the first part of the process. We have

$$T_1 = 546 \text{ and } \frac{v_1}{v_2} = \frac{1}{5}$$

$\therefore$ taking $\gamma = 1{\cdot}406$,

$$T_2 = \left(\frac{1}{5}\right)^{0\cdot 406} \times 546 = 284.$$

The work done *by* the gas

$$= \frac{R\,(T_1 - T_2)}{\gamma - 1} = \frac{96 \times 262}{0{\cdot}406} = 62{,}000 \text{ ft. lbs.}$$

No heat is taken in or rejected since the process is adiabatic.

For the second stage we use § 19. We now have $T_1 = 284$.

The work done *by* the gas

$$= RT_1 \log_e \frac{v_2}{v_1} = 96 \times 284 \times \log_e \frac{1}{5}$$

$$= -\,27{,}300 \times 1{\cdot}61 = -\,44{,}000 \text{ ft. lbs.}$$

*i.e.* the work done *on* the gas $= +\,44{,}000$ ft. lbs. There is no increase of internal energy, and the thermal equivalent of this ($= 44{,}000 \div 1400 = 31{\cdot}4$ Th. Units) is rejected.

In the last stage, no work is done since the volume remains constant. The heat supplied is

$$k_v\,(546 - 284) = 0{\cdot}169 \times 262 = 44{\cdot}3 \text{ Th. Units.}$$

Summing up we have

| | 1st Stage. | 2nd Stage. | 3rd Stage. | Total. |
|---|---|---|---|---|
| Heat taken in (Th. Units) | 0 | $-31{\cdot}4$ | $44{\cdot}3$ | $12{\cdot}9$ |
| Work done (ft. lbs.) ... | 62,000 | $-44{,}000$ | 0 | 18,000 (= 12·9 Th. Units.) |

The last column serves as a check on the work; for always: heat received − heat rejected = work done.

**4.** Unit mass of a perfect gas is made to expand in such a way that its volume is at any instant proportional to its pressure, the initial values being $v_1$ and $p_1$ respectively. If the pressure be changed from $p$ to $p + dp$, find the values of $dE$ and $dQ$ in terms of $p$ and $dp$. (Intercoll. Exam. Cambridge, 1909.)

We have (§14)

$$dE = k_v\, dT \qquad \text{(i.)}$$

Also, since $pv = RT$, we have

$$\frac{dp}{p} + \frac{dv}{v} = \frac{dT}{T}$$

$$\therefore \qquad dT = T\left(\frac{dp}{p} + \frac{dv}{v}\right) = \frac{pv}{R}\left(\frac{dp}{p} + \frac{dv}{v}\right).$$

But, by the conditions of the problem,

$$\frac{dv}{v} = \frac{dp}{p} \text{ and } \frac{v}{v_1} = \frac{p}{p_1}$$

$$\therefore \qquad dT = 2\frac{pv}{R}\cdot\frac{dp}{p} = 2.\frac{v_1}{p_1}.\frac{pdp}{R}.$$

Hence, from (i.)

$$dE = 2\frac{k_v}{R}\cdot\frac{v_1}{p_1}\cdot pdp \qquad \text{(ii.)}$$

Again,

$$\begin{aligned} dQ &= dE + dW \\ &= dE + \frac{p.dv}{J} \\ &= 2\frac{k_v}{R}\cdot\frac{v_1}{p_1}\, pdp. + \frac{v_1}{p_1}.\frac{pdp}{J} \\ &= \frac{v_1}{p_1}\cdot p.dp.\left(2\frac{k_v}{R} + \frac{1}{J}\right) \qquad \text{(iii.)} \end{aligned}$$

(ii.) and (iii.) are the expressions required.

**5.** Two lbs. of air at 17°C. are compressed suddenly from an absolute pressure of 15 lbs./in.$^2$ to an absolute pressure of 45 lbs./in.$^2$ Taking $R = 95{\cdot}7$ and $\gamma = 1{\cdot}4$, find the volume before compression, and the volume and temperature immediately after. (Special Exam. Cambridge, 1911.)

**6.** Twenty cubic feet of gas at 15°C. and atmospheric pressure are suddenly compressed (the law being $pv^{1{\cdot}4} =$ const.) to one-fourth the volume. Find the pressure and temperature attained. The gas is then allowed to cool to its original temperature: find the final pressure and the work, in foot pounds, lost in compression. (Special Exam. Cambridge, 1913.)

**7.** One pound of air, at a pressure of 15 lbs./in.$^2$ and temperature 15·5°C., is compressed adiabatically to a pressure of 30 lbs./in.$^2$, and at this pressure is cooled to its initial temperature. It is then further compressed adiabatically to a pressure of 60 lbs./in.$^2$, and again cooled at this pressure to its initial temperature. Determine the temperature and volume at the end of each compression, and sketch the $pv$ diagram. (Intercoll. Exam. Cambridge, 1909.)

**8.** A cartridge containing 4 lbs. of air at 1000 lbs./in.$^2$ by gauge and 15°C. is placed in the chamber of a gun behind a light frictionless piston fitting the bore of the gun. The cartridge is perforated and the piston just reaches the muzzle of the gun. Calculate the mean temperature of the air and the volume of the gun, on the assumption that the air absorbs no heat from the walls of the gun. Atmospheric pressure $= 14{\cdot}7$ lbs./in.$^2$ (Mech. Sc. Trip. 1912.)

**9.** Air is compressed adiabatically into a vessel of $V$ cubic ft. capacity to $m$ times the atmospheric density. Show that, if $p$ be the atmospheric pressure, the work expended is

$$pV\left(\frac{m^\gamma - \gamma m}{\gamma - 1} + 1\right)$$

foot-pounds. (Trin. Coll., Cambridge, Schol. Exam. 1905.)

## GENERAL PRINCIPLES II.

21. In the theory of heat engines we deal with processes in which a substance or system changes its state. These processes may be reversible or irreversible. A **reversible process** is one in which all the parts of the acting system can be brought back to their original condition without leaving any change in other bodies or systems. All other processes are irreversible. That a process should be irreversible it is not enough that it should not be directly reversible; it must be impossible, even with the assistance of all the agents in nature, to restore everywhere the exact initial conditions, leaving no changes in the agents used to effect the reversal.

*Examples of reversible processes:* The motion of a projectile in an unresisting medium; adiabatic and isothermal expansion.

*Examples of irreversible processes:* The motion of a projectile in a resisting medium; the unresisted expansion of a gas; generation of heat by friction; transference of heat by conduction.

22. **Second Law of Thermodynamics:** Heat cannot pass from one body to a warmer body without some compensating transformation taking place (without work being done).

23. **Carnot's Cycle.** We require a standard to indicate the maximum amount of work that we can hope to get out of an ideal engine using a given quantity of heat.

We have (1) A "source" of heat consisting of an infinite body at temperature $T_1$ absolute.

(2) A "cooler" in the form of an infinite body at temperature $T_2$, and $T_2 < T_1$.

(3) An engine using any material for its working substance.

Since the source and the cooler are infinite, their temperatures will be unaltered. however much heat we take from the one or give to the other. We perform the following series of operations which together constitute Carnot's cycle:

(1) Let the working substance be at $T_1$ and take heat $Q^1$ from the source. This is isothermal.

(2) Take away the source and let the substance pass adiabatically to the lower temperature $T_2$.

(3) Apply the cooler and let the working substance give out $Q_2$ to it at $T_2$. This again is isothermal.

(4) Take away the cooler and make the working substance pass adiabatically back to $T_1$, so that it is in the same state as when it started, and so has gone through a cycle.

Now the engine has received $Q_1$, given back $Q_2$, and has none left. Hence an amount of heat $Q_1 - Q_2$ has gone out of existence, and therefore an equal amount of work (measured in heat units) has been done, and we have

$$\mathbf{W} = \mathbf{Q_1} - \mathbf{Q_2} \text{ heat units.}$$

$$\text{the efficiency} = \frac{\text{energy obtained in work}}{\text{energy supplied in heat}}$$

$$= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}.$$

24. **Reversibility of Carnot's Cycle.** The following things have happened: The source has lost $Q_1$, the cooler has gained $Q_2$, the working substance has done work $= Q_1 - Q_2$ through the engine. Now let the cooler give up the heat $Q_2$ to the substance at temperature $T_2$, and let the engine do work $Q_1 - Q_2$ on the substance raising its temperature to $T_1$, so that it now has heat $Q_1$, which it can give up to the source at $T_1$. Thus everything is left as it started, and no changes have occurred outside the system. Hence the cycle is reversible.

Note that here all the reception and rejection of heat takes place at the highest and lowest temperatures respectively, and this is an essential condition for the reversibility. If at any time, during the supply of heat, there were a difference of temperature between the source and the working substance, the transference of heat from the former to the latter would involve conduction which is an irreversible process. Similarly with the rejection of the heat.

25. Is this the best we can do? **To prove that any reversible engine is more efficient than any irreversible one,** and hence that **all engines working on any reversible cycle have the same efficiency.**

Take a reversible engine $A$, and an irreversible one $B$.

If possible let $B$ be more efficient than $A$.

Let $B$ work direct and drive $A$ reversed.

Let $B$ receive heat $Q_1$ at the high temperature $T_1$ and produce work $W$.

Let $A$ receive work $W$ and produce heat $Q_2$ also at $T_1$.

If $A$ worked direct it would do work $W$ when supplied with $Q_2$, whereas $B$ requires $Q_1$.

But, $B$ is more efficient than $A$.

$$\therefore\ Q_1 < Q_2 \text{ or } Q_2 > Q_1.$$

The nett effect of the combination is that no external work is done, the source gives up $Q_1$ and receives $Q_2$ which is $> Q_1$, which is impossible by the second law.

$$\therefore\ B \text{ cannot be more efficient than } A.$$

This argument is in no wise upset if we substitute 'reversible' for 'irreversible' in the case of $B$. Our result then is that no one reversible engine $B$ can be more efficient than *any* other reversible engine $A$. Therefore all reversible engines have the same efficiency.

26. **To prove that the efficiency of any reversible engine is** $1 - \frac{T_2}{T_1}$, where $T_1$ is the temperature at which the heat is supplied, and $T_2$ is the lowest available temperature, *i.e.* the temperature of the cooler.

Since all reversible engines have the same efficiency it suffices to find the efficiency of any one. Take one which works with a perfect gas as its working substance. Carnot's Cycle is then represented graphically as in Fig. 1, where, along :—

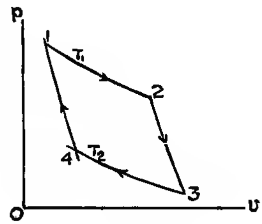



Fig. 1.

1—2, heat is supplied at constant temperature $T_1$.

2—3 is adiabatic expansion.

3—4, heat is rejected at temperature $T_2$.

4—1 is adiabatic compression to temperature $T_1$.

$$Q_1 = \text{heat received along } 1\text{—}2$$

$$= RT_1 \log_e \frac{v_2}{v_1} \qquad \text{by §18.}$$

$$Q_2 = \text{heat rejected along } 3\text{—}4$$

$$= RT_2 \log_e \frac{v_3}{v_4}.$$

No heat is received or rejected along 2—3 or 4—1.

Now $p_3v_3 = RT_2 = p_4v_4$,

$$\therefore \frac{v_3}{v_4} = \frac{p_4}{p_3}$$

$$= \frac{\dfrac{p_1v_1^{\gamma}}{v_4^{\gamma}}}{\dfrac{p_2v_2^{\gamma}}{v_3^{\gamma}}} = \frac{p_1v_1^{\gamma}v_3^{\gamma}}{p_2v_2^{\gamma}v_4^{\gamma}},$$

$$\therefore \frac{v_3^{1-\gamma}}{v_4^{1-\gamma}} = \frac{p_1v^{\gamma}}{p_2v_2^{\gamma}} = \frac{v_2}{v_1} \cdot \frac{v_1^{\gamma}}{v_2^{\gamma}} = \frac{v_2^{1-\gamma}}{v_1^{1-\gamma}},$$

$$\therefore \frac{v_3}{v_4} = \frac{v_2}{v_1}.$$

Hence
$$\frac{Q_2}{Q_1} = \frac{RT_2 \log_e \frac{v_3}{v_4}}{RT_1 \log_e \frac{v_2}{v_1}} = \frac{T_2}{T_1}.$$

$\therefore$ the efficiency (§23) $= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$, which applies to any engine working on any reversible cycle.

27. **Available Energy.** By §§23—26 the maximum amount of work obtainable from a quantity of heat $Q_1$ drawn from a source at temperature $T_1$ when the temperature of the cooler is $T_2$ is

$$W = Q_1 - Q_2 = Q_1\left(1 - \frac{Q_2}{Q_1}\right) = Q_1\left(1 - \frac{T_2}{T_1}\right).$$

In other words,

of the heat $Q_1$ $\begin{cases} Q_1\left(1 - \frac{T_2}{T_1}\right) \text{ is } \textit{available energy}, \\ \text{the remainder, } Q_1\frac{T_2}{T_1} = Q_2, \text{ is } \textit{unavailable.} \end{cases}$

If we had another cooler capable of receiving heat continuously at a lower temperature $T_0$, we could get out of $Q_2$ an amount of work or available energy

$$= Q_2\left(1 - \frac{T_0}{T_2}\right),$$

the unavailable energy then

$$= Q_2\frac{T_0}{T_2} = Q_1\frac{T_2}{T_1}\frac{T_0}{T_2} = Q_1\frac{T_0}{T_1} = T_0\frac{Q_1}{T_1} = Q_0 \text{ say.}$$

Hence the unavailable energy associated with a given quantity of heat is (i) proportional to the lowest absolute temperature available in the cooler, (ii) inversely proportional to the temperature of the body which the heat is entering.

**Def :** The **Available Energy** of a system subject to given external conditions is the maximum amount of

mechanical work theoretically obtainable from the system without violating the given conditions.*

Hence $\frac{Q}{T}$ may be regarded as a measure of the unavailability or the factor which only has to be multiplied by any assumed auxiliary temperature $T_0$ in order to give the quantity of unavailable energy relative to that temperature.

This factor is called *entropy*.

28. Hence, immediately, we have two **definitions of entropy:**

(i.) If a system at temperature $T$ receive a small quantity of heat $dQ$ the quotient $\frac{dQ}{T}$ is called the increase of entropy of the system arising from this cause.

If $A$ and $B$ denote two different states of a system which are capable of being connected together by a continuous series of reversible transformations the change of entropy of the system in passing from $A$ to $B$ is defined as

$$\Sigma \int_A^B \frac{dQ}{T}$$

where the $\Sigma$ extends to all parts of the system and the $\int$ is taken along the reversible series of transformations referred to.

(ii.) If the unavailable energy of a system with reference to an auxiliary medium of temperature $T_0$ undergo any

*Note the difference between total energy measured above a given zero and available energy when the same zero point is taken, *e.g.* the total potential energy of a mass of 10 pounds 10ft. above the earth's surface is 100 ft.-lbs., the earth's surface being taken as position of zero energy; now interpose a table underneath the weight and 5ft. high, this does not alter the potential energy of the weight but only $10 \times 5 = 50$ ft.-lbs. are now available since the weight can only fall as far as the table. In the above the whole heat energy measured above 0° C is $Q_1$, but only $Q_1 (1 - T_0/T_1)$ is available.

positive or negative increase, and if this increase be divided by $T_0$ the quotient is called the increase of entropy of the system.

When (i.) is given it must be given as precisely as above.

(i.) is inapplicable to most irreversible phenomena.

(ii.) makes no restrictions as to the nature of the transformations which take place, and holds for irreversible changes.

28A. **To prove that for maximum efficiency an engine must take in all its heat supply at the highest temperature, and give out all its rejected heat at the lowest temperature.** If a quantity of heat $Q$ be received at temperature $T$, when the lowest available temperature is $T_0$, the unavailable energy is $\frac{Q}{T}T_0$. Hence, if an engine receive part of its heat at temperatures below $T$, this unavailable energy is increased, and we cannot get so much work out of the total heat supply. Similarly the available energy is reduced if some of the heat rejection take place at temperatures above the lowest available. Hence an engine can only have its maximum efficiency when the whole of the heat supply is taken in at the highest temperature, and all its rejected heat is given out at the lowest temperature.

29. **To prove that** there is no total change of entropy when a substance or system is taken through a reversible cycle, *i.e.* that

$$\int \frac{dQ}{T} = 0$$

for the whole cycle.

The unavailable energy cannot change unless the physical state changes.

∴, by Def. (ii.) of §28, the entropy cannot change unless the physical state changes.

But a necessary condition for a cycle is that the physical state should be same at the end as at the beginning.

$\therefore$ There can be no change of entropy.

$\therefore$ the sum of all the changes of entropy, *i.e.* the sum of all the $\frac{dQ}{T}$'s, is zero,

$$\therefore \int \frac{dQ}{T} = 0, \text{ taken round the cycle.}$$

30. **To prove that the value of $\int_1^2 \frac{dQ}{T}$ is the same for all reversible paths connecting the states 1 and 2.**

Suppose a reversible cycle represented by a curve 1 A 2 B 1 (Fig. 2). Then by §29

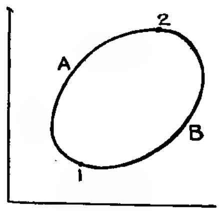



Fig. 2.

$$\int_{1A2B1} \frac{dQ}{T} = 0,$$

$$\therefore \quad \int_{1A2} \frac{dQ}{T} + \int_{2B1} \frac{dQ}{T} = 0,$$

$$\therefore \quad \int_{1A2} \frac{dQ}{T} = - \int_{2B1} \frac{dQ}{T} = \int_{1B2} \frac{dQ}{T}.$$

But 1A2 and 1B2 are *any* two reversible paths connecting 1 and 2,

$\therefore \quad \int \frac{dQ}{T}$ is constant for all reversible paths connecting 1 and 2, *i.e.* the change of entropy in passing from one state to another depends only on the two states, and not on the mode of passage.

Thus the entropy of a substance under given conditions depends only on the state of the substance and not at all on its past history.

We are not concerned here with the entropy of irreversible changes, but in these cases there is a loss of availability, and consequently a gain of unavailability of energy, *i.e.* an increase of entropy.

## EXAMPLES.

**1.** Find the maximum amount of work which can theoretically be obtained from 1 lb. of water at 100°C. if all the heat wasted may be given out at 15°C. Assume the sp. ht. of water to be constant and equal to unity. (Intercoll. Exam. Cambridge, 1911.)

The heat contained in the water = 100 Th. Units.

The portion which must be wasted is

$$100 \times \frac{288}{373} = 77{\cdot}2$$

∴ The work which could theoretically be obtained is $22{\cdot}8 \times 1400 = 32{,}000$ ft. lbs.

**2.** A heat engine is used to drive a reversed heat engine as a refrigerator. The heat engine takes in its heat at absolute temperature $T_1$ and the refrigerator at an absolute temperature $T_2$. Both engines reject heat at $T_0$. Shew that the maximum amount of heat which can be extracted by the refrigerator per unit of heat taken in by the heat engine is $\dfrac{T_2(T_1 - T_0)}{T_1(T_0 - T_2)}$ (Intercoll. Exam. Cambridge, 1913.)

If the engine receive heat $Q_1$ it will do an amount of work $W_1 = Q_1\left(1 - \dfrac{T_0}{T_1}\right)$.

If the refrigerator, working as an engine, receives heat $Q_0$ at temperature $T_0$ and reject it at $T_2$, it will do work

$$W_2 = Q_0\left(1 - \frac{T_2}{T_0}\right)$$

and reject heat $= Q_0 \dfrac{T_2}{T_0}$

Conversely, working as a refrigerator, if it receive work $W_2$ and heat $Q_0 \dfrac{T_2}{T_0}$ it will give out heat $= Q_0$.

If $W_2$ comes from the first engine, it must $= W_1$.

$$\therefore \qquad Q_0\left(1-\frac{T_2}{T_0}\right)=Q_1\left(1-\frac{T}{T_1}\right)$$

$$\therefore \; Q_0=\frac{T_0\,(T_1-T_0)}{T_1\,(T_0-T_2)}\,Q_1.$$

The heat which the refrigerator has extracted at temperature $T_2$ is $Q_0\,\frac{T_2}{T_0}$, which $=\frac{T_2(T_1-T_0)}{T_1(T_0-T_2)}\,Q_1$.

The amount per unit of heat supplied to the first engine is

$$\frac{T_2(T_1-T_0)}{T_1(T_0-T_2)}$$

**3.** The specific heat of a substance at an absolute temperature $T$ may be taken to be given by $a+bT$, where $a$ and $b$ are constants. Shew that, if the heat from 1 lb. of the substance be used as efficiently as possible in doing work, the quantity of heat wasted will be equal to

$$T_0\left\{b(T_1-T_0)+a\log_e\frac{T_1}{T_0}\right\}.$$

$T_1$ being the initial temperature of the substance, and $T_0$ the lowest available temperature.

Of a quantity of heat, $dQ$, taken in at temperature $T$, an amount must be wasted equal to

$$\frac{T_0}{T}\,dQ=\frac{T_0}{T}(a+bT)\,dT.$$

The body can go on giving out heat until its temperature is $T_0$. Hence the total waste will be

$$\int_{T_0}^{T_1}\frac{T_0}{T}(a+bT)\,dT$$

$$=T_0\int_{T_0}^{T_1}\left(\frac{a}{T}+b\right)dT$$

$$=T_0\left\{a\log_e\frac{T_1}{T_0}+b\,(T_1-T_0)\right\}.$$

**4.** A heat engine takes in 500 Th. Units at a temperature of 150°C. and rejects its heat to a body whose temperature rises 1°C. for every 5 Th. Units of heat absorbed. If the temperature of the body be initially 15°C. shew that the maximum work which the engine can do is 115 Th. Units, and that the final temperature of the body will then be 92°C. (Mech. Sc. Trip., 1913.)

## PERFECT GASES II.

31. We use $\phi$ to denote entropy.

By definition (i.) § 28,

$$d\phi = \frac{dQ}{T},$$

also $dQ = dE + dW$.................... by §8.

$$= dE + \frac{pdv}{J} \text{ ....................by §15.}$$

for a small change.

$$\therefore \quad d\phi = \frac{dE}{T} + \frac{pdv}{JT},$$

$$\therefore \quad dE = Td\phi - \frac{1}{J}pdv,$$

$\therefore$, for a reversible cycle, since $E$ must be same at the end as at the beginning,

$$O = \int dE = \int Td\phi - \frac{1}{J}\int pdv,$$

or $$J\int Td\phi = \int pdv.$$

We can represent a cycle by plotting $T$ and $\phi$ or by plotting $p$ and $v$, and this equation shows that the area (expressed in ft.-lb.) of the temperature-entropy diagram equals the area of the pressure-volume diagram, and therefore represents the work done in the cycle.

32. In the T-$\phi$ diagram, an adiabatic is represented by $\phi$ = constant, since dQ = 0, *i.e.* by a line parallel to the T axis, and an isothermal is T = constant and so parallel to the $\phi$ axis.

*e.g.* A Carnot cycle consists of two adiabatics and two isothermals, and so is drawn as in Fig. 3, where the numbers

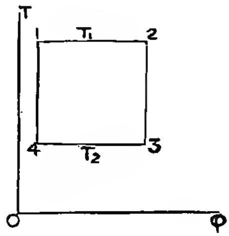
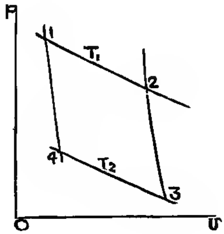



Fig. 3




correspond in each diagram.

33. **To find the change of entropy of a perfect gas** in changing from $(p_1v_1T_1)$ to $(p_2v_2T_2)$.

For small changes,

$$dW = \frac{pdv}{J} \quad \text{.................... by §15}$$

$$dE = k_v dT \quad \text{.................... by §14. ii.}$$

$$\therefore \quad d\phi = \frac{dQ}{T} = \frac{dE+dW}{T} = k_v\frac{dT}{T} + \frac{1}{J}\frac{pdv}{T},$$

but $$pv = RT \text{ and } \therefore \frac{p}{T} = \frac{R}{v}$$

$$\therefore \quad d\phi = k_v\frac{dT}{T} + \frac{R}{J}\cdot\frac{dv}{v} \quad \text{.......................... (i.)}$$

also $$\log p + \log v = \log R + \log T$$

$$\therefore \quad \frac{dp}{p} + \frac{dv}{v} = \frac{dT}{T} \quad \text{.................................. (ii.)}$$

By (i.) and (ii.) we can express $\phi$ in terms of $p$ and $v$, $v$ and $T$, or $T$ and $p$, by eliminating the third letter.

(a) Eliminate $T$: $d\phi = k_v\left(\frac{dp}{p} + \frac{dv}{v}\right) + \frac{R}{J}\frac{dv}{v},$

$$= k_v\left(\frac{dp}{p} + \frac{dv}{v}\right) + (k_p - k_v)\frac{dv}{v};$$

$$= k_v\frac{dp}{p} + k_p\frac{dv}{v}.$$

$$\therefore \quad \phi_2 - \phi_1 = \mathbf{k_v \log_e \frac{p_2}{p_1} + k_p \log_e \frac{v_2}{v_1}} \quad \text{.........} \quad \text{(A)}$$

(b) Eliminate $v$: $d\phi = k_v\frac{dT}{T} + (k_p - k_v)\left(\frac{dT}{T} - \frac{dp}{p}\right)$

$$= k_p\frac{dT}{T} - (k_p - k_v)\frac{dp}{p},$$

$$\therefore \quad \phi_2 - \phi_1 = k_p \log_e\frac{T_2}{T_1} - (k_p - k_v)\log_e\frac{p_2}{p_1} \text{ .........(B)}$$

(c) From (i)

$$\phi_2 - \phi_1 = k_v \log_e\frac{T_2}{T_1} + (k_p - k_v)\log_e\frac{v_2}{v_1} \text{ .........(C)}$$

34. **To calculate the entropy of a gas in a given state** ($p\ v\ T$).

Entropy can only be measured above some arbitrary state of zero entropy. We take 0° C (273° abs.), 14·7 lbs/in² as state of zero entropy.

Hence $T_1 = 273$, $p_1 = 14{\cdot}7$ lbs/in².

Find the corresponding value of $v_1$; put $\phi_1 = 0$.

Then $\qquad \phi = k_v \log_e\frac{p}{14{\cdot}7} + k_p \log_e\frac{v}{v_1}.$

We shall now consider a few standard cycles which have been devised, and find the efficiency of, and the work obtainable from, engines working on these cycles.

**35. The Stirling Cycle.** (Two isothermals, two lines of constant volume.) It is distinguished by the use of a *regenerator*, *i.e.* an apparatus capable of storing heat temporarily and giving it up again. The cycle is (see Fig. 4):

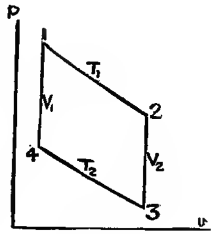



Fig. 4.

1—2. Air at $T_1$ expands isothermally from $v_1$ to $v_2$, taking in heat and doing work.

$$Q_1 = \text{heat taken in per lb.} = RT_1 \log_e \frac{v_2}{v_1}.$$

2—3. The air passes through the regenerator at volume $v_2$; temperature reduced to $T_2$. Heat given to the regenerator $= k_v(T_1 - T_2)$.

3—4. The air is compressed isothermally to volume $v_1$, giving up heat.

$$Q_2 = \text{heat rejected} = RT_2 \log_e \frac{v_2}{v_1}.$$

4—1. The air passes through the regenerator, and picks up the heat it left behind during 2—3, viz. $k_v(T_1 - T_2)$, its temperature becoming $T_1$.

$$W = \text{work done} = Q_1 - Q_2 = R(T_1 - T_2) \log_e \frac{v_2}{v_1}$$

$$\therefore \text{ efficiency} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}.$$

*Effect of Inefficiency of the Regenerator.* If the regenerator have an efficiency of, say, 60%, instead of 100%, as assumed above, only ·6 of the heat left behind in 2—3 will be picked up in 4—1.

36. **The Joule Cycle.** (Two constant pressure lines and two adiabatics.) Fig. 5.

By §19,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4}$$

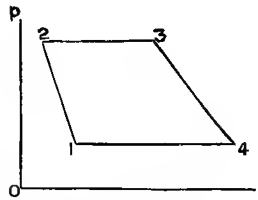



Fig. 5.

1—2. Adiabatic compression. No heat received or rejected.

2—3. $Q_1$ = heat received = $k_p\ (T_3 - T_2)$ per lb.

3—4. Adiabatic expansion. No heat received or rejected.

4—1. $Q_2$ = heat rejected = $k_p\ (T_4 - T_1)$ per lb.

$W$ = work done per lb. = $Q_1 - Q_2$.

$$\therefore \quad \text{Efficiency} = \frac{k_p(T_3 - T_2) - k_p(T_4 - T_1)}{k_p(T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2},$$

$$= 1 - \frac{T_4}{T_3} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1}$$

$$= 1 - \left(\frac{1}{r}\right)^{\gamma-1},$$

where $$r = \frac{v_1}{v_2} = \frac{v_4}{v_3}.$$

37. **The Otto Cycle.** (Two constant-volume lines and two adiabatics.) Fig. 6.

We have by §19

$$\frac{T_1}{T_4} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \frac{T_2}{T_3}.$$

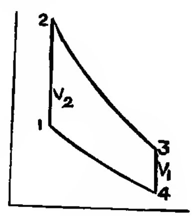



Fig. 6.

4—1. Adiabatic compression. No heat received.

1—2. $Q_1$ = heat received
= $k_v\ (T_2 - T_1)$ per lb.

2—3. Adiabatic expansion. No heat received or rejected.

3—4. $Q_2$ = heat rejected = $k_v (T_3 - T_4)$ per lb.

$W$ = work done = $Q_1 - Q_2$.

∴ efficiency

$$= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_3 - T_4}{T_2 - T_1} = 1 - \frac{T_3}{T_2} = 1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1}$$

$$= 1 - \left(\frac{1}{r}\right)^{\gamma-1}$$

## EXAMPLES.

**1.** Calculate the change of entropy when the state of 1 lb. of air is changed from 15 lbs./in.$^2$, 15°C., to 2250 lbs./in$^2$., 10°C.

We shall use equation (B) of § 33.

$$\phi_2 - \phi_1 = k_p \log_e \frac{283}{288} - (k_p - k_v) \log_e \frac{2250}{15}$$

$$= 0{\cdot}237 \log_e 0{\cdot}983 - 0{\cdot}068 \log_e 150$$

$$= -\ 0{\cdot}345.$$

**2.** In Example 3, p. 15, calculate the entropy of the air in its initial state, and at the end of the adiabatic expansion, also at the end of the isothermal compression.

**3.** Find the efficiency of a Stirling Engine if the regenerator efficiency is 65%.

**4.** Draw the entropy-temperature diagram for Example 7 on p. 17.

**5.** In a Joule cycle the upper and lower pressures are 100 and 15 lbs./in.$^2$, the temperature at the beginning of adiabatic compression is 10°C. Calculate the efficiency and draw the temperature-entropy diagram if the ratio of expansion at constant pressure be four.

38. **Heat of formation** under constant pressure. We take as the state of zero heat the state of the substance at 0° C, and under zero pressure. The total heat of a vapour in a given state will be the heat required to bring it from 0° C, and zero pressure, to the given state, and consists of the increase of internal energy + the work done (§3), there being supposed no change of kinetic energy.

We shall, throughout, be dealing with one pound of the substance, unless otherwise stated.

Suppose we have a vertical cylinder, whose cross section is 1 sq. ft., fitted with a piston weighing $p$ lbs. Then the pressure exerted by the piston, on whatever may be underneath, is always $p$ lbs./sq. ft.

We require to find the heat of formation of a vapour under this pressure $p$, *i.e.* the heat required to bring 1lb. of liquid, at 0° C, zero pressure, into the cylinder and from it into vapour.

Let $V$ = volume, in cubic ft., of 1lb. of vapour at pressure $p$.

$\omega$ = volume, in cubic ft., of 1lb. of liquid at pressure $p$.

$t$ = temperature, °C.

(1) *To force the liquid into the cylinder.* Originally the piston is at the bottom, and volume underneath is zero. When the liquid has been pushed in the volume underneath is $\omega$ ft$^3$; the cross section is 1 ft$^2$. Therefore the piston bas been raised $\omega$ ft.

$$\therefore \text{work done} = p\omega \text{ ft.-lbs.}$$

(2) *To raise the temperature of the liquid to* t° C. Taking the sp. ht. of the liquid as constant and $= \sigma$

Heat required = mass × sp. ht. × $t$.

$= \sigma t.$

Work done = 0, if we neglect expansion of the liquid.

$\therefore I_w$ = total heat of liquid at t° C, pressure $p$.

$$= \sigma t + \frac{p\omega}{J}.$$

(3) *To evaporate the liquid.* When the temperature reaches the temperature of evaporation, the liquid begins to evaporate, and the piston is raised. Thus there is an increase of internal energy and work is done. At any instant let there be $q$ lbs. of vapour and $(1 - q)$ lbs. of liquid.

Then $q$ is called the **dryness fraction.**

Let $L$ = the latent heat of the vapour,

= heat required to evaporate 1lb. = work done + increase of internal energy.

Then to evaporate $q$ lbs. will require heat $qL$.

The volume of vapour will be $qV$: an increase of $q(V - \omega)$.

$\therefore$ the work done $= qp(V - \omega)$.

Let $\rho$ = increase of internal energy on evaporating 1 lb.

Then $q\rho$ = ,, ,, ,, ,, $q$ lbs.

Thus, $qL$ = heat added in evaporating $q$ lbs.

= increase of internal energy + work done

$= q\rho + qp(V - \omega)/J$

$\therefore$ For complete evaporation

$$L = \rho + p(V - \omega)/J,$$

$\therefore I$ = total heat of vapour at dryness $q$

$$= I_w + qL = I_w + q\left\{\rho + p(V - \omega)/J\right\}$$

$I_s$ = total heat of dry saturated ($q = 1$) vapour,

$$= I_w + L = I_w + \left\{\rho + p\frac{(V - \omega)}{J}\right\}.$$

The internal energy $= \sigma t + q\rho$.........if wet.

or $E = \sigma t + \rho$......... if dry.

(4) *To superheat the vapour.* If we go on adding heat after the liquid is all evaporated the vapour becomes superheated.

Let $K_p$ = sp. ht. of vapour.

$t'$ = temperature to which we superheat it.

$t' - t$ is called the degrees of superheat.

We shall suppose $K_p$ constant.

Heat added in superheating $= K_p (t' - t)$.

$\therefore I'_s$ = total heat of superheated vapour

$$= I_s + K_p (t' - t)$$

$$= I_w + L + K_p (t' - t).$$

These results hold for any vapour; in dealing with water and steam take $\sigma = 1$ for calculations.

39. Summary of results.

$\mathbf{I_w}$ = total heat of liquid $= \sigma \mathbf{t} + \dfrac{\mathbf{p}\omega}{\mathbf{J}}$

$\rho$ = internal energy added in vaporization.

$\mathbf{L}$ = latent heat $= \rho + \mathbf{p}(\mathbf{V} - \omega)/\mathbf{J}$

$\mathbf{I_s}$ = total heat of dry saturated vapour

$$= \mathbf{I_w} + \mathbf{L} = \mathbf{E_s} + \frac{\mathbf{pV}}{\mathbf{J}}.$$

$\mathbf{E_s}$ = total internal energy of vapour $\rho + \sigma \mathbf{t}$.

$\mathbf{I}$ = total heat of wet vapour (dryness fraction $q$).

$$= \mathbf{I_w} + \mathbf{qL}$$

$$= \mathbf{qI_s} + (\mathbf{1} - \mathbf{q})\,\mathbf{I_w}.$$

Similarly

$\mathbf{E}$ = internal energy of wet vapour

$$= \mathbf{qE_s} + (\mathbf{1} - \mathbf{q})\,\mathbf{E_w}$$

Where $\mathbf{E_w}$ = internal energy of liquid $= \sigma \mathbf{t}$ if the sp. heat be constant.

40. **Formation of vapour under any conditions.** We always have

$$\mathbf{I} = \mathbf{E} + \frac{\mathbf{pV}}{\mathbf{J}}$$

Where $E$ = total gain of internal energy, and depends only on the initial and final states, and not on the manner of getting from one to the other.

$pV$ = the work done by the expanding fluid.

41. **Entropy of Vapours.** We seek the entropy of a vapour, wet, dry, or superheated, and to find it we again consider the cylinder process of §38.

(1) In heating the liquid.

$dQ$ = heat required to raise temperature of 1lb. of liquid by $dT$

$$= \sigma dT$$

$$\therefore \quad d\phi = \frac{dQ}{T} = \frac{\sigma dT}{T}.$$

$$\therefore \ \phi - \phi_0 = \int_{T_0}^{T} \frac{\sigma dT}{T} = \sigma \log_e \frac{T}{T_0}.$$

if $\sigma$ be supposed constant.

If we take $\phi_0$, the entropy in the initial state, as zero, and $T_0 = 273$,

$$\phi_w = \text{entropy of liquid} = \sigma \log_e \frac{T}{273}.$$

Hence the $\phi - T$ diagram for heating water is a logarithmic curve, as shown by O A in Fig. 7.

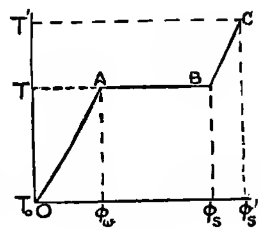



Fig. 7.

(2) When the liquid reaches the temperature of evaporation for the given pressure, evaporation commences, the temperature remaining constant. Therefore in the $\phi - T$ diagram we go along a horizontal line A B.

To evaporate to dryness $q$ requires heat $qL$.

$$\therefore \text{ the increase of entropy } = \frac{qL}{T}.$$

$\therefore$ at any instant during evaporation

$$\phi = \sigma \log_e \frac{T}{273} + \frac{qL}{T}$$

where $T$ = saturation temperature.

When evaporation is complete we have

$$\phi_s = \text{entropy of dry saturated vapour}$$

$$= \sigma \log_e \frac{T}{273} + \frac{L}{T}.$$

We thus arrive at point B (Fig. 7).

For the wet vapour we can write, then,

$$\phi = q\phi_s + (1-q)\,\phi_w$$

(3) In superheating, to increase the temperature by $dT$ requires heat

$$dQ = K_p\, dT.$$

$$\therefore\ d\phi = K_p \frac{dT}{T}$$

$\therefore$ total increase of $\phi$ in superheating to a temperature $t'$ ($T'$ absolute)

$$= \int_T^{T'} K_p \frac{dT}{T} = K_p \log_e \frac{T'}{T},$$

if $K_p$ be supposed constant.

$\therefore \phi'_s$ = entropy of superheated vapour

$$= \sigma \log_e \frac{T}{273} + \frac{L}{T} + K_p \log_e \frac{T'}{T}.$$

On the $\phi - T$ diagram we go up another logarithmic curve BC (Fig. 7).

42. For every different pressure there is a different temperature of evaporation, giving a point A on the line OA, and a corresponding point B. If we plot the points A and B for different pressures we get two curves $AA^1$, $BB^1$. (Fig. 8).

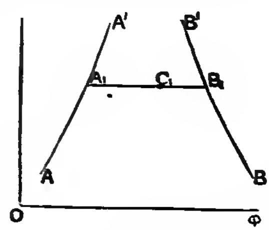


Fig. 8.

43. In §41 we saw that to evaporate to dryness $q$ we go along A B a distance $\frac{qL}{T}$, and for complete evaporation we go a distance $\frac{L}{T}$. Hence if we have

steam in any condition, and know its entropy and temperature, *i.e.* its "state point" $C_1$ say, we can find its dryness by drawing the line of evaporation, $A_1 B_1$, through $C_1$ and then

$$\mathbf{q} = \frac{\mathbf{A_1 C_1}}{\mathbf{A_1 B_1}} = \frac{\phi_c - \phi_{w1}}{\phi_{s1} - \phi_{w1}}.$$

## 44. Specific Heat of Superheated Vapours.

We have taken $K_p$ as constant, but this is not correct. Hence to calculate the entropy of a superheated vapour we must find the mean value of $K_p$ over the range $t$ to $t'$, thus:

$$\text{We have } I'_s = K_p(t' - t) + I_s$$

$$\therefore \quad K_p = \frac{I'_s - I_s}{t' - t}$$

and the correct value of $I'_s$ (calculated by other means) is given in the book of tables.

## 45. Adiabatic Expansion of Vapours.

To find the adiabatic equation.

Let a vapour, in the state denoted by suffix 1, expand adiabatically to the state denoted by 2. The condition of adiabatic expansion is $\phi$ = constant.

$$\therefore \phi_2 = \phi_1,$$

whether the vapour be wet, dry or superheated. If wet we have

$$\sigma \log_e \frac{T_2}{273} + \frac{q_2 L_2}{T_2} = \sigma \log_e \frac{T_1}{273} + \frac{q_1 L_1}{T_1}$$

$$\therefore \quad \frac{\mathbf{q_2 L_2}}{\mathbf{T_2}} = \frac{\mathbf{q_1 L_1}}{\mathbf{T_1}} + \sigma \mathbf{\log_e} \frac{\mathbf{T_1}}{\mathbf{T_2}},$$

Whence the dryness, $q_2$, at the end of expansion can be calculated.

This is a most important equation, and should always be used in dealing with problems on the adiabatic expansion or compression of vapours.

**For steam** $\sigma = 1$, and

$$\frac{q_2 L_2}{T_2} = \frac{q_1 L_1}{T_1} + \log_e \frac{T_1}{T_2}.$$

If the steam start superheated the value of $\phi_1$ contains an extra term (§41.3) which slightly modifies the equation.

45A. **Condensation of Vapours at Constant Volume.** If a given quantity ($w$ lbs.) of steam or other vapour be enclosed in a given volume $v$ and be allowed to cool, it will condense, and the dryness at any pressure is found thus :

At the pressure at which the dryness is required, find, from the tables, the volume of 1 lb. of dry saturated vapour, $V$ say. Then the volume of $w$ lbs. would be $wV$.

The total real volume of liquid and vapour $= v$.

Then, if dryness $= q$,

wt. of liquid $= (1 - q)w$ lbs. Vol. $= (1 - q)w\ \omega$

wt. of vapour $= qw$ lbs. Vol. $= qwV$

Hence

$$(1 - q)w\ \omega + qwV = v.$$

Hence find $q$.

If $\omega$ is negligible (as in case of steam) $qwV = v$

$$\therefore \quad \mathbf{q} = \frac{v}{wV} = \frac{\textbf{actual volume}}{\textbf{vol. it would have if dry}}.$$

46. **Passage of Vapours through nozzles; throttling.** When steam or any other vapour is forced through a throttle-valve, nozzle or other small orifice, it becomes drier, or, if it be already dry it becomes superheated.

**To prove that, in these cases,** $I$ **is constant,** there being supposed no change of kinetic energy, the process also being supposed adiabatic.

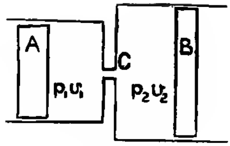


Fig 9.

Suppose the vapour is pushed through an orifice C by one piston A, while a piston B recedes as A approaches the orifice.

On the A side one pound of vapour has internal energy $E_1$; as A approaches the hole C it does work $p_1 V_1$ on the vapour. When the fluid gets the other side of C it does work $p_2 V_2$ on B.

$\therefore$ nett work done on fluid $= p_1 V_1 - p_2 V_2$.

$$\therefore \quad E_2 = E_1 + \frac{p_1 V_1 - p_2 V_2}{J},$$

there being no heat supplied from without.

$$\therefore \quad E_2 + \frac{p_2 V_2}{J} = E_1 + \frac{p_1 V_1}{J},$$

or $$I_2 = I_1.*$$

47. Hence we have two fundamental conditions to remember in dealing with vapours:

(1) **Adiabatic expansion,** $\phi_2 = \phi_1$.

(2) **Adiabatic throttling or passage through nozzles, $I_2 = I_1$.**

The actual form of these equations depends on whether the vapour be wet, dry, or superheated at the beginning. The general instructions are: write down the equation, and insert suitable values of $\phi$ (§41) or $I$ (§39) as the case may be.

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* It must be noted that, although this process is adiabatic, $\phi$ is not constant, but increases. This is because throttling is an irreversible process.

## EXAMPLES.

**1.** One pound of dry steam at a pressure of 120 lbs./in.$^2$ abs. is allowed to condense at constant volume until its pressure falls to 20 lbs./in.$^2$ abs. What is then the dryness fraction, and how much heat has been rejected?

What would be the final state of the steam if it expanded adiabatically through the same range? (Intercoll. Exam. Cambridge, 1907.)

(i.) Condensing at constant volume: from the Tables we find:

Vol. of 1 lb. of dry steam at 120 lbs./in.$^2$ = 3·746 ft.$^3$

" " " " 20 " = 20·06 ft.$^3$

The weight of dry steam at the lower pressure which would occupy 3·746 ft.$^3$

$$= \frac{3{\cdot}746}{20{\cdot}06} = 0{\cdot}187 \text{ lbs.}$$

∴ neglecting the volume of the condensed steam, the dryness fraction is 0·187 (*cf.* § 45.A.)

In the process, no external work is done, and therefore (§ 8), the heat rejected is equal to the loss of internal energy.

From the Tables we find:

$$\text{at } 120 \text{ lbs./in.}^2 \quad E_s = 618$$

$$\text{,, } 20 \text{ lbs./in.}^2 \quad E_s = 601{\cdot}9, \; E_w = 109{\cdot}4$$

Hence (§ 39), at 20 lbs./in.$^2$, with $q = 0{\cdot}187$, we have

$$E = 0{\cdot}187 \times 601{\cdot}9 + 0{\cdot}813 \times 109{\cdot}4$$
$$= 112{\cdot}6 + 89 = 201{\cdot}6.$$

The heat rejected is, then

$$618 - 201{\cdot}6 = 416{\cdot}4 \text{ Th. Units.}$$

(ii.) Adiabatic expansion: we shall find the final dryness fraction from the condition that the entropy remains constant (§ 45).

At 120 lbs./in.$^2$ $\phi = \phi_s = 1{\cdot}596$.

,, 20 lbs/in.$^2$ $\phi_s = 1{\cdot}735$, and $\phi_w = 0{\cdot}337$.

$\therefore$ if the dryness be $q$, we have

$$\phi = 1{\cdot}735\, q + 0{\cdot}337\,(1 - q) = 0{\cdot}337 + 1{\cdot}398\, q.$$

Then $0{\cdot}337 + 1{\cdot}398\, q = 1{\cdot}596$.

$$\therefore \qquad q = \frac{1{\cdot}259}{1{\cdot}398} = 0{\cdot}90.$$

Or we can proceed thus, using the full equation of §45 :

At 120 lbs./in.² $\quad L = 490{\cdot}2, \quad T = 444{\cdot}6$

,, 20 lbs./in.² $\quad L = 533{\cdot}7, \quad T = 381{\cdot}8$

$q$ is then given by

$$\frac{533{\cdot}7\, q}{381{\cdot}8} = \frac{490{\cdot}2}{444{\cdot}6} + \log_e \frac{444{\cdot}6}{381{\cdot}8}$$

$$1{\cdot}4\, q = 1{\cdot}105 + \log_e 1{\cdot}163 = 1{\cdot}256$$

$$q = 0{\cdot}896.$$

The slight difference between the two values obtained for $q$ is due to taking the specific heat of water as constant and equal to unity. The first method is the more accurate and is arithmetically more simple.

**2.** Ten pounds of water at 50°C. are heated and become superheated steam at a temperature of 350°C., and a pressure of 200 lbs./in.² Calculate the change of entropy. The steam is then expanded adiabatically ; to what pressure must it fall before becoming dry saturated steam ? (Intercoll. Exam. Cambridge, 1913.)

$\phi_1$ = entropy of water at 50°C. = 0·1685.

$\phi_s$ = ,, dry steam at 200 lbs./in.² = 1·555.

$t$ = temperature ,, ,, = 194·2°C.

mean* specific heat at 200 lbs./in.² over the range required = 0·548.

Then (§ 41)

$$\phi'_s = \phi_s + K_p \log_e \frac{T^1}{T}$$

$$= 1{\cdot}555 + 0{\cdot}548 \log_e \frac{350 + 273}{194{\cdot}2 + 273}$$

$$= 1{\cdot}711$$

*Tabulated in "The New Steam Tables" by Smith & Warren (Constable).

From the Tables we see that the steam will be just dry at 27 lbs./in.$^2$

**3.** Steam is being blown out of a boiler in which the pressure is 200 lbs./in.$^2$ abs. into air where the pressure is 15 lbs./in.$^2$ abs. The kinetic energy of the steam is very small, and there is no loss of heat by conduction. If the issuing steam be just dry, find its initial dryness fraction. (Mech. Sc. Trip., 1910.)

The condition which obtains is that the Total Heat remains constant (§ 46).

Let $q$ be the dryness fraction of the steam in the boiler. Then, from the Tables,

$$I_1 = 669{\cdot}8\,q + 197{\cdot}5\,(1-q) = 197{\cdot}5 + 472{\cdot}3\,q.$$

$$I_2 = 639{\cdot}9 \text{ (since } q = 1).$$

Hence we have

$$197{\cdot}5 + 472{\cdot}3\,q = 639{\cdot}9$$

$$\therefore \qquad q = \frac{442{\cdot}4}{472{\cdot}3} = 0{\cdot}936.$$

**4.** Two boilers, whose volumes are in the ratio of 2 to 3, are under steam at pressures 200 and 150 lbs./in.$^2$ abs. respectively. The first boiler is $\frac{1}{4}$ full of water, and the second is half full. A connection is opened between them, and the resulting pressure is 180 lbs./in.$^2$ Assuming that no steam has escaped, find the total quantity of heat, per unit of total volume, rejected from, or supplied to, the whole system during the equalization of pressures. (Mech. Sc. Trip., 1913.)

Let the volumes be $2V$ and $3V$ cubic feet.

Then the volumes of water are $0{\cdot}5V$ and $1{\cdot}5V$

and ,, ,, steam ,, $1{\cdot}5V$ ,, $1{\cdot}5V$

∴ ,, weights of water ,, $31{\cdot}25V$ ,, $93{\cdot}75V$ lbs.

and ,, steam (from tables) ,, $\frac{1{\cdot}5}{2{\cdot}316}V$ ,, $\frac{1{\cdot}5}{3{\cdot}027}$

∴ ,, total weights are $31{\cdot}9V$ and $94{\cdot}24\,V$ respectively, or $126{\cdot}1\,V$ for the two boilers.

The total value is $5V$.

Let $xV$ be the volume of water, the weight of which will be $62{\cdot}5\ xV$.

The volume of steam at 180 lbs./in.$^2$ will be $(5-x)\ V$.

The pressure is 180 lbs./in.$^2$ and from the tables we find that the volume of 1 lb. of steam is 2·558 ft.$^3$

Therefore the weight of steam is $\dfrac{(5-x)V}{2{\cdot}558}$

Thus the total weight is

$$62{\cdot}5\,Vx + \frac{(5-x)\ V}{2{\cdot}558}.$$

But this must be the same as the initial weight.

$$\therefore \qquad 62{\cdot}5x + \frac{5-x}{2{\cdot}558} = 126{\cdot}1$$

which gives $\qquad x = 2$ nearly.

To find the heat supplied or lost, we apply the energy equation of § 8 to the whole system. No external work has been done, and therefore the heat supplied = the increase of internal energy. For the first boiler the initial internal energy is (from the tables)

$$V(31{\cdot}25 \times 197{\cdot}2 + 0{\cdot}65 \times 622{\cdot}1) = 6555V,$$

for the second it is

$$V(93{\cdot}75 \times 183{\cdot}6 + 0{\cdot}494 \times 619{\cdot}8) = 17{,}500V,$$

making a total of $24{,}055V$.

In the final stage the weight of water $= 125V$, and the weight of steam $= 1{\cdot}17V$ lbs.

The internal energy is

$$V(125 \times 192 + 1{\cdot}17 \times 621{\cdot}2) = 24{,}730V.$$

Thus there is an increase of internal energy equal to $675\ V$.

$\therefore$ The heat supplied per unit of total volume

$$= \frac{675V}{5V} = 135 \text{ Th. Units.}$$

**5.** In order to warm the feed-water of a boiler, saturated steam at 150 lbs./in.$^2$ abs. is blown into it. How much steam must be used per pound of water in order to raise the temperature from 20°C. to 60°C. ? (Special Exam. Cambridge, 1912.)

**6.** A cylinder fitted with a piston contains 1 lb. of steam and water at 120°C. Find how much steam and how much water is present when the volume is 10 cub. ft. (Special Exam. Cambridge, 1912.)

**7.** A boiler evaporates 2,000 lbs. of water per hour, the temperature of the feed being 50°C., and the pressure in the boiler 180 lbs./in.$^2$ abs. The efficiency of the boiler is 70%, and the calorific value of the coal used is 8000 C. Th. Units per lb. How much coal is used per hour ?

**8.** A pound of steam expands from 100 lbs./in.$^2$ abs. to 20 lbs./in.$^2$ abs., being kept just dry all the time. Plot a curve to show how the volume varies with the pressure and find the work done. (Special Exam. Cambridge, 1913.)

**9.** Steam at a pressure of 250 lbs./in.$^2$ abs., dryness fraction 0·96, expands adiabatically to a pressure of 20 lbs./in.$^2$ abs. What will be the dryness fraction ? (Intercoll. Exam. Cambridge, 1904.)

**10.** Steam at 150 lbs./in$^2$. abs., dryness fraction 0·9, expands (*a*) at constant volume, (*b*) adiabatically, (*c*) at constant total heat, (*d*) at constant dryness. The final pressure is 40 lbs./in.$^2$ Plot the $\phi - T$ diagram in each case.

**11.** One pound of dry steam at 60 lbs./in.$^2$ is condensed at constant volume. Find the dryness fraction when the pressure is 20 lbs./in.$^2$

**12.** A boiler evaporates water to steam of dryness 0·95 at a pressure of 150 lbs./in.$^2$ Determine the efficiency of the boiler if 20,000 lbs. of water are evaporated, per ton of coal,

from feed-water at 80°C. The calorific value of the coal is 8000 Th. Units per lb. (Intercoll. Exam. Cambridge, 1913.)

**13.** Calculate the maximum amount of work which can theoretically be obtained from one pound of steam at 100 lbs./in.$^2$ with an initial dryness of 0·9. The lowest available temperature is 15°C. (Schol. Exam. Cambridge, 1913.)

**14.** A boiler contains 3000 lbs. of water and steam at 200 lbs./in$^2$. abs., half the total volume being water. How much steam must be blown off to reduce the pressure to 190 lbs./in.$^2$?

**15.** Half a pound of wet steam enclosed in a cylinder occupies a volume of 1 ft.$^3$ at a pressure of 50 lbs./in.$^2$ Heat is supplied to it until the pressure is raised to 100 lbs./in.$^2$, the volume being kept constant. It is then allowed to expand to double the volume, the pressure remaining constant. Find the heat supplied in each process and the external work done. (Intercoll. Exam. Cambridge, 1914.)

**16.** Dry saturated steam at 100 lbs./in.$^2$ abs. expands adiabatically to 3 lbs./in.$^2$

If the expansion be by throttling, what energy is available per pound when the steam expands—

(*a*) from a vessel of constant volume ?

(*b*) from a vessel in which constant pressure is maintained?

If the expansion were behind a piston and perfectly reversible, find the dryness fraction of the steam at the lower pressure. (R.N.C. Greenwich, 1911-12.)

**48. The Rankine Cycle.** The ideal cycle for any heat engine is the Carnot Cycle, but this is impracticable. In the steam engine, steam passes from the boiler to the cylinder at (theoretically) constant temperature, and at the same time steam is formed in the boiler. This is all isothermal and corresponds to the first step of Carnot. Then the steam expands, as nearly as possible, adiabatically, after which it gives up some of its heat isothermally by being condensed in

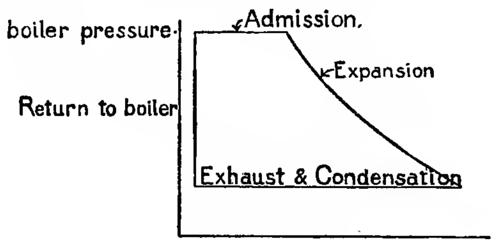



Fig. 10.

a condenser. So far this is the same as Carnot's cycle; but the fourth step, adiabatic compression, is impracticable. The best we can do is to make the condensation complete and pump back the condensed water to the boiler. This cycle is called the *Rankine Cycle,* and is the standard to which we try to make our engine approach. The p.v. diagram is shown in Fig. 10.

The diagram of a real engine differs from this because (1) The steam does not enter the cylinder at boiler pressure, because of the passage through pipes, valves, etc. (2) The expansion is neither complete nor adiabatic. (3) The valves do not open or shut instantaneously, which rounds off the corners of the diagram. (4) There is a certain amount of compression at the end of the exhaust stroke, because the valve closes before the end of the stroke.

The entropy diagram is shown in Fig. 11. Along AB the water is heated from hotwell temperature $T_2$ to boiler temperature $T_1$; along BC evaporation takes place, which is followed by adiabatic expansion down a vertical line: CD if the steam start dry, EF if it start wet, GH if it start superheated. HA is the line of condensation.

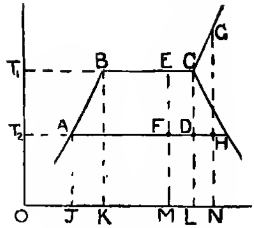



Fig. 11.

Heat taken in during heating of water = ABKJ

" " evaporation = BCLK

or BEMK

" " superheating (if any) = CGNL

Heat rejected during condensation

= ADLJ (starting dry)

AFMJ ( " wet)

or AHNJ ( " superheated)

as the case may be.

The work done = area between AB, BC, the expansion line, and HA.

49. **To find the work done in Rankine Cycle per lb. of steam.** We have

$$dQ = dE + dW,$$

$$= dE + \frac{p\,dV}{J}.$$

$$= dE + \frac{d(pV)}{J} - \frac{V\,dp}{J},$$

$$= dI - \frac{V\,dp}{J}.$$

In an adiabatic process $dQ = 0$.

$$\therefore \quad dI = \frac{Vdp}{J},$$

$$\therefore \quad I_1 - I_2 = \frac{1}{J}\int_{p_2}^{p_1} Vdp,$$

= the area ABCD (Fig. 12)

= the work done.

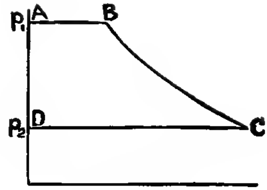


Fig. 12.

Hence

$$\mathbf{W = I_1 - I_2} \quad \text{.........................................(i.)}$$

$$= I_{w1} + L_1 - (I_{w2} + q_2L_2),$$

$$= t_1 + \frac{p_1\omega}{J} + L_1 - \left(t_2 + \frac{p_2\omega}{J} + q_2L_2\right),$$

$$= (t_1 - t_2) + L_1 - q_2L_2 + \frac{p_1 - p_2}{J}\omega.$$

$$= (T_1 - T_2) + L_1 - T_2\left(\frac{L_1}{T_1} + \log_e \frac{T_1}{T_2}\right) + \frac{p_1 - p_2}{J}\omega,$$

$$= (T_1 - T_2)\left(1 + \frac{L_1}{T_1}\right) - T_2 \log_e \frac{T_1}{T_2} + \frac{p_1 - p_2}{J}\omega.$$

Of this work, the part $(p_1 - p_2)\omega/J$ is used in pumping the water from the condenser to the boiler.

Hence the external work

$$= \mathbf{(T_1 - T_2)\left(1 + \frac{L_1}{T_1}\right) - T_2 \log_e \frac{T_1}{T_2}} \quad \text{.........(ii.)}$$

$$= \mathbf{I_1 - I_2},$$

if we neglect the $\omega$ term, which is very small indeed.

If proper values be given to $I_1$ and $I_2$ to suit the case, the formula $W = I_1 - I_2$ applies whether the steam be supplied wet, dry, or superheated, but (ii.) only applies when the steam starts dry.

50. **Efficiency of Rankine Cycle.** One lb. of water enters the boiler at $t_2$ and is turned into steam. The water contains heat $I_{w2}$ when it enters the boiler, and the heat of the steam formed is $I_1$.

$$\therefore \text{ Heat required per lb.} = I_1 - I_{w2}.$$

$$\therefore \quad \text{efficiency} = \frac{W}{I_1 - I_{w2}} = \frac{I_1 - I_2}{I_1 - I_{w2}}.$$

By substituting the above value of $W$, we have

$$\text{efficiency} = \frac{(T_1 - T_2)\left(1 + \frac{L_1}{T_1}\right) - T_2 \log_e \frac{T_1}{T_2}}{I_1 - I_{w2}}.$$

51. **To find the number of pounds of steam used** by a Rankine Engine per I.H.P. hour.

Let H = H.P. output.

1 I.H.P. hour = 33000 × 60 ft. lbs.

Work done per lb. of steam $= W = I_1 - I_2$.

$$\therefore \quad \text{steam required} = \frac{33000 \times 60\text{H}}{I_1 - I_2} \text{lbs. per hour.}$$

52. **Mollier's Diagram of Entropy and Total Heat.** This affords quite the simplest method of dealing with questions of the adiabatic expansion of steam, and other problems. The diagram is drawn with the horizontal axis as axis of $\phi$ and the vertical axis as axis of $I$. A sketch is shown in Fig. 13, and diagrams drawn to scale are obtainable.* On the

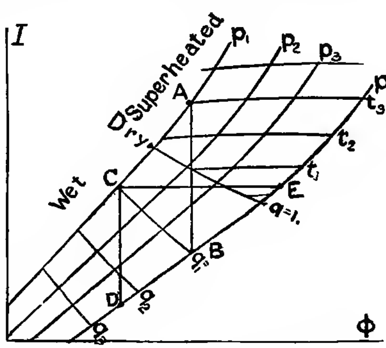


Fig. 13.

* From the Publishers, price 1s. nett.

diagram are drawn lines of constant pressure such as $p_1$ $p_2$, etc., lines of constant dryness, $q_1$ $q_2$, etc., in the wet region, and lines of constant temperature, $t_1$ $t_2$, etc., in the superheat region.

In adiabatic processes $\phi$ is constant, and so the adiabatics are vertical, while lines of constant $I$ are horizontal.

53. **To use Mollier's Diagram.** (1) *For Rankine's Cycle.* Take the pressure line corresponding to the initial pressure, and follow it up until the proper temperature line is reached, at A say, if the steam be superheated, or the proper dryness line, at C, if it be wet. This gives the starting point; read off the value of $I_1$. Now follow a vertical line downwards till the line of condenser pressure is reached at B or D. At this point read $I_2$, and the dryness can also be read if desired. Then the vertical distance between the two points gives at once the work done, measured in heat units, by the engine per lb. of steam used, since C D or A B, according to conditions, is equal to the heat drop, $I_1 - I_2$.

(2) *For steam passing through a throttling valve or other small orifice.* By §46 we have $I_2 = I_1$. Hence, to use the diagram to find the condition of the steam after it has passed through the orifice: find the point corresponding to the initial state of the steam, *e.g.* C, and go along a line of constant $I$, *i.e.* a horizontal, until the line of low pressure is reached, at E say, then the condition of the steam can be read off at once.

The diagram is very useful, and saves a lot of time, and the student is advised to obtain one and familiarize himself with its use.

54. **To draw the Saturation Curve on the Indicator Diagram.** The object of this is to examine the state of the steam throughout the stroke.

(i.) In the test of the engine the amount of steam used per hour or per minute will be measured.

From this the steam used per stroke can be calculated when the r.p.m. are known.

(ii.) Calculate the cushion steam:—the piston does not travel to the end of the cylinder, and the volume between the end of the cylinder and the piston at the same end of its stroke is called the *clearance* volume. In this space a certain amount of steam is enclosed, which is called the *cushion steam.* To find its

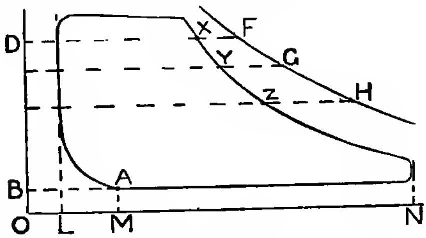



Fig. 14.

amount:—Mark the point A, on the indicator card, where compression begins (where the line begins to go round the corner). The whole stroke is LN; hence the volume of the steam shut up in the cylinder $= v$ $=$ clearance vol. (supposed known) $+ \dfrac{LM}{LN} \times$ vol. swept out by piston in its stroke.

Take the steam as dry, and read its pressure OB. In the tables find the volume of one lb. of steam at this pressure, $= V$ say. Then

$$\text{wt. of cushion steam} = \frac{v}{V} \text{ lbs.}$$

(iii.) Then the total wt. of steam present during expansion $= w =$ wt. used per stroke $+$ wt. of cushion steam.

(iv. For a series of points x, y, z, . . . on the expansion line calculate the actual volume of the steam as

shown for finding volume of cushion steam. Suppose, at x, vol. $= v_1$. Find the volume of one lb. at the pressure at x from the tables, $= V_1$ say. Then the volume of $w$ lbs. (wt. actually in the cyclinder) $= wV_1$. This is the volume the steam would have if it were dry; the actual volume is $v_1$. Hence the dryness $= q_1 = \frac{v_1}{wV_1}$. The real volume is represented by DX. Hence, the volume it would have if dry is represented by DF, where $\frac{DX}{DF} = q_1$. In this way find the points FGH . . . , and the curve through them is the saturation curve.

55. **To show the exchange of heat between steam and cylinder walls** on the $\phi - T$ diagram. (Fig. 15.) From the saturation curve drawn on the indicator diagram (§54), find the dryness of the steam at different points x, y, z (Fig. 14). Draw on the $\phi$-T diagram the water line $FA_1$ and the steam line $GB_1$. From the tables find the temperatures $T_1$ $T_2$, etc., corresponding to the pressures at x, y, z . . . . and draw horizontal lines $A_1B_1$, etc., at heights $T_1$ $T_2$ . . . . Suppose $A_1$ $B_1$ correspond to X. Divide $A_1$ $B_1$ at $C_1$ so that

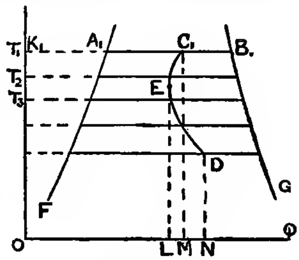


Fig. 15.

$$\frac{A_1C_1}{A_1B_1} = \text{dryness at X.}$$

In this way draw the curve $C_1$ E D.

If the expansion were adiabatic it would follow $C_1$ M. From $C_1$ to E the dryness decreases, so the steam is giving heat to the metal, and amount given up $= C_1$ E L M. From E to D, $q$ increases, and the steam receives heat E L N D from the metal.

## EXAMPLES.

**1.** A Rankine engine takes dry steam at 150 lbs./in.$^2$ abs., and exhausts at 16 lbs./in.$^2$ abs. Find the steam consumption per I.H.P. per hour.

Find the tables $I_1 = I_{s1} = 666{\cdot}7$.

$$\phi_1 = \phi_{s1} = 1{\cdot}578.$$

Let $q$ be the dryness fraction at the end of expansion.

At 16 lbs./in.$^2$ we have

$$I_w = 103{\cdot}0 \qquad \phi_w = 0{\cdot}320$$
$$I_s = 640{\cdot}6 \qquad \phi_s = 1{\cdot}752$$
$$\phi_2 = 1{\cdot}752\,q + 0{\cdot}32\,(1 - q) = 0{\cdot}32 + 1{\cdot}432\,q.$$

Also, $\phi_2 = \phi_1$

$\therefore\ 1{\cdot}432\,q + 0{\cdot}320 = 1{\cdot}578$

$$\therefore \qquad q = \frac{1{\cdot}258}{1{\cdot}432} = 0{\cdot}88.$$

Hence

$$I_2 = 0{\cdot}88 \times 640{\cdot}6 + 0{\cdot}12 \times 103$$
$$= 577.$$

The work done per lb. of steam $= I_1 - I_2 = 666{\cdot}7 - 577$

$= 89{\cdot}7$ Th. Units.

$= 125{,}600$ ft. lbs.

Hence the steam required per 1 H.P. per hour

$$= \frac{33{,}000 \times 60}{125{,}600} = 15{\cdot}8 \text{ lbs.}$$

**2.** An engine using steam at a pressure of 150 lbs./in.$^2$ abs., and of dryness 0·95, indicates 750 H.P., and is found to use 280 lbs. of steam per minute. If the pressure in the condenser is 1·5 lb./in.$^2$ abs., find the thermal efficiency of the engine, and determine what would have been the consumption of the engine if it had worked upon the Rankine cycle between the same pressures. (Intercoll. Exam., Cambridge, 1912.)

At 150 lbs./in.$^2$ $I_w = 183{\cdot}9$, $I_s = 666{\cdot}7$.

Hence $I_1 = 0{\cdot}95 \times 666{\cdot}7 + 0{\cdot}05 \times 183{\cdot}9 = 642$.

At 1·5 lbs./in.$^2$ $I_w = 46{\cdot}7$.

Hence the heat supplied per lb. of steam $= 642 - 46{\cdot}7$
$= 595{\cdot}3$

$\therefore$ ,, ,, ,, ,, minute $= 595{\cdot}3 \times 280$
$= 167{,}000$ Th. Units.

The work done per minute $= 750 \times 33{,}000$ ft. lbs.
$= 17{,}700$ Th. Units

$$\therefore \text{ the efficiency} = \frac{17{,}700}{167{,}000} = 10{\cdot}6\ \%.$$

On the Rankine cycle :

$$\phi_1 = 0{\cdot}95 \times 1{\cdot}578 + 0{\cdot}05 \times 0{\cdot}515 = 1{\cdot}526$$

$$\phi_2 = 1{\cdot}94q_2 + 0{\cdot}158\,(1 - q_2) = 0{\cdot}158 + 1{\cdot}782q_2.$$

Also $\phi_1 = \phi_2$

$$\therefore \qquad q_2 = \frac{1{\cdot}368}{1{\cdot}782} = 0{\cdot}77.$$

Hence $I_2 = 0{\cdot}77 \times 616{\cdot}4 + 0{\cdot}23 \times 46{\cdot}7 = 486.$

The work done per lb. of steam

$= I_1 - I_2 = 156$ Th. Units
$= 218{,}000$ ft. lbs.

$\therefore$ The steam consumption per minute

$$= \frac{750 \times 33{,}000}{218{,}000} = 113 \text{ lbs.}$$

**3.** From an engine trial we have the following data: cylinder diameter 12″: stroke 26″: R.P.M. 100: steam used in both ends of the cylinder 18 lbs. per min.: clearance volume 8% of stroke volume: barometer 29·6″, equal to 14·5 lbs. per sq. in. The length of the mean indicator diagram is 73 mm., and at distances $x$ from the admission end of the diagram the following pressures (above or below atmospheric) are scaled off:

Point $A$ in compression line, $x = 3$ mm., $p = -6$ lbs. sq. in.
Point $B$ in expansion line, $x = 34$ mm., $p = 15$ lbs. sq. in.

Assuming that the clearance steam is dry during compression, find its amount, and determine the whole amount of steam present during expansion and its state at the point *B*. (Intercoll. Exam. Cambridge, 1909.)

The steam used per stroke $= \dfrac{18}{200} = 0{\cdot}09$ lbs.

The stroke volume $= \dfrac{\pi \times 36 \times 26}{1728} = 1{\cdot}7$ ft.$^3$

The clearance volume $= 0{\cdot}08 \times 1{\cdot}7 = 0{\cdot}136$ ft.$^3$

The linear scale of the indicator diagram is

$$1 \text{ mm.} = \frac{1{\cdot}7}{73}$$
$$= 0{\cdot}0233 \text{ ft.}^3$$

At A, the volume of the steam

$$= 0{\cdot}136 + 0{\cdot}0699 = 0{\cdot}2059 \text{ ft.}^3,$$

and the pressure $= 14{\cdot}5 - 6 = 8{\cdot}5$ lbs./in.$^2$ abs.

At this pressure the volume of 1 lb. of dry steam $= 44{\cdot}8$ ft.$^3$

The weight of cushion steam $= \dfrac{0{\cdot}2059}{44{\cdot}8} = 0{\cdot}0046$ lb.

The total wt. of steam present during expansion

$$= 0{\cdot}09 + 0{\cdot}0046 = 0{\cdot}0946 \text{ lbs.}$$

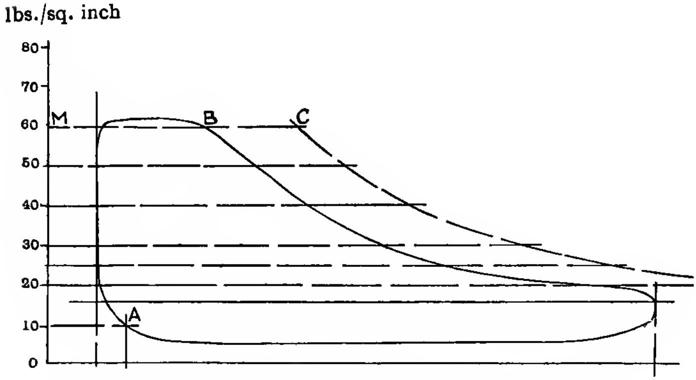
At B, the volume $= 0{\cdot}136 + 34 \times 0.0233 = 0{\cdot}928$ ft.$^3$, and the pressure $= 14{\cdot}5 + 15 = 29{\cdot}5$ lbs./in.$^2$ abs.

The volume of 1 lb. of dry steam at this pressure $= 14$ ft.$^3$

If the steam in the cylinder were dry its volume would be $14 \times 0{\cdot}0946 = 1{\cdot}325$ ft.$^3$, but its actual volume is $0{\cdot}928$ ft.$^3$

$\therefore$ the dryness fraction at B $= \dfrac{0{\cdot}928}{1{\cdot}325} = 0{\cdot}7$.

**4.** Fig. 16 is the indicator-diagram for a double-acting steam engine, and shows the necessary particulars. Draw the saturation curve to scale, deduce the $T - \phi$ diagram, and deduce the heat exchanges between the steam and cylinder. (Mech. Sc. Trip., 1910.)




| | | | |
|---|---|---|---|
| Diameter cylinder | = 1 foot | Steam per min. | = 17·3 lbs. |
| Stroke | = 2 feet | Clearance | = 9 % |
| R.P.M. | = 96 | | |

The diagram applies to both ends of the cylinder. Neglect the area of piston rod.

Proceeding as in Ex. 3, we find:

Steam used per stroke = 0·09 lbs.

Stroke volume = 1·57 ft.$^3$

(Thence 1″ on diagram = 0·358 ft.$^3$)

Clearance ,, = 0·1415 ft.$^3$

Taking the point A, we find $p = 10$ lbs./in.$^2$, $V = 0{\cdot}217$ ft$^3$.

Wt. of cushion steam = 0·00825 lbs.

Wt. ,, steam during expansion = 0·09825 lbs.

The remainder of the calculations are most conveniently done in a table, thus:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| $p$ lbs./in.$^2$ | $v$ | $V$ ft.$^3$ | $q=\frac{v}{V}$ | T | $q\phi_s$ | $\overline{1-q}\ \phi_w$ | $\phi$ | $d\phi$ | mean $T$ | $dQ = Td\phi$ |
| 60 | 0·444 | 0·705 | 0·630 | 417·7 | 1·040 | 0·158 | 1·198 | | | |
| | | | | | | | | +·096 | 414·5 | 39·2 |
| 50 | 0·590 | 0·836 | 0·705 | 411·3 | 1·172 | 0·122 | 1·294 | | | |
| | | | | | | | | ·026 | 407·5 | 10·6 |
| 40 | 0·741 | 1·03 | 0·720 | 403·6 | 1·21 | 0·110 | 1·320 | | | |
| | | | | | | | | −·011 | 398·9 | −4·4 |
| 30 | 0·95 | 1·35 | 0·704 | 394·3 | 1·20 | 0·109 | 1·309 | | | |
| | | | | | | | | −·002 | 391·4 | −0·8 |
| 25 | 1·12 | 1·60 | 0·700 | 388·5 | 1·20 | 0·107 | 1·307 | | | |
| | | | | | | | | +·114 | 385·1 | +43·9 |
| 20 | 1·525 | 1·97 | 0·775 | 381·8 | 1·345 | 0·076 | 1·421 | | | |

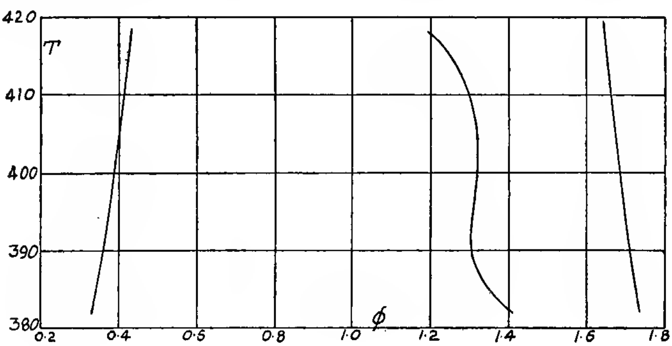
In column 2, $v$ is found by measuring the abscissa, on the diagram, of the point in question (*e.g.* $B$), and multiplying by the scale ($1'' = 0{\cdot}358$ ft.)$^3$

In column 3, $V$, the volume the steam would occupy if dry, is found by multiplying the specific volume (given in the tables) by the weight of steam in the cylinder (0·09825 lb.).

The coördinates of the saturation curve are obtained from column 3 by dividing $V$ by the scale

$$(e.g.\ MC = \frac{\cdot 705}{\cdot 358} = 1{\cdot}97 \text{ ins.})$$

The next step is to calculate $\phi$ for each pressure (columns 6-8). The $\phi - T$ diagram is then drawn. (Fig. 16A.)



From this we calculate the exchange of heat as explained in § 55. It can be done approximately as in columns 10 and 11. The heat received by the steam is $\int T d\phi$, and the value of the integral is found roughly by taking the graph as straight from point to point.

While the pressure is falling from 60 to 40 lbs./in.$^2$, the steam is receiving heat from the cylinder and getting drier. From 40 to 25 lbs./in.$^2$ the steam suffers a slight loss of heat, but after that continues to receive heat to the end.

The figures in column 11 give the gain or loss of heat per pound of steam.

**5.** Find the indicated work per pound of steam on the Rankine cycle, when the steam-chest gauge reads 120 lbs. per sq. in., the vacuum gauge 28″ and the barometer 30″ (Intercoll. Exam., Cambridge, 1911.)

**6.** The pressures and total volumes for two points on the expansion curve of a steam engine indicator card are as follows:

$p_1 = 80 \qquad p_2 = 20$ lbs. per sq. in. abs.

$v_1 = 0{\cdot}185 \qquad v_2 = 0{\cdot}72$ cubic feet.

The mass of steam expanding is estimated at 0·0375 lbs. Find the dryness of the steam at the two points, and the net heat supplied to it during the expansion from $v_1$ to $v_2$. The expansion curve may be assumed of the form $pv^n$ = constant. (Intercoll. Exam. Cambridge, 1911.)

**7.** From an indicator diagram, the pressure at 0·7 of the stroke is found to be 25 lbs. per sq. in. abs., whilst, when the cushioning begins, at 0·9 of the return stroke, the absolute pressure is 4 lbs. per sq. in. The clearance volume is 10% of the volume swept out by the piston, the latter being 1·5 cub. ft. The engine is double acting and uses 0·18 lb. of steam per double stroke. Estimate the dryness of the steam at 0·7 of the stroke. (Trin. Coll., Cambridge, 1914).

**8.** Use the Mollier $\phi - I$ diagram to solve the following problems :—

(i.) Steam of dryness 0·85 expands adiabatically and reversibly from a pressure of 200 lbs. per sq. in. to a pressure of 2 lbs. per sq. in. Find the heat drop and the final dryness.

(ii.) Steam expands at constant dryness, 0·85, between the above limits of pressure. Find the heat drop and increase of entropy.

(iii.) Dry steam is throttled from the 200 lbs. per sq. in. pressure to 2 lbs. per sq. in. Find the change in entropy and the final state of the steam. (Mech. Sc. Trip., 1916.)

## SURFACE CONDENSERS.

§ 55A. If no air is present :

heat given up by steam = heat taken in by cooling water.

$$I_2 = w(t_1 - t_2)$$

where $w$ = $wt$ of cooling water per lb. of steam

$t_1$ = temp. of water entering condenser

$t_2$ = temp. of water leaving condenser

$I_2$ = total heat of steam entering condenser.

If air is present in the exhaust steam, let

$p_1$ = pressure of vapour, assuming no air is present

$p_2$ = pressure of air, assuming no vapour is present,

then total pressure $= p = p_1 + p_2$.

The steam tables give volume of 1 lb. of vapour, so if we know the actual volume we can find the weight.

The volume of 1 lb. of dry air is given by $p_2 v = RT$.

## EXAMPLES.

**1.** The volume of a condenser, which contains 0·1 lb. of air with the steam, is 98 ft.$^3$ The temperature is 45°C., and

there is some water at the bottom of the condenser. Find the weight of vapour and the pressure in the condenser.

From the tables the pressure of the vapour

$$= 1{\cdot}382 \text{ lbs./in.}^2$$

From the tables vol. per lb. $= 246$ ft.$^3$

$$\therefore \text{ weight of vapour present } = \frac{98}{246} = 0{\cdot}398 \text{ lb.}$$

The vol. of the air per lb. $= 980$ ft.$^3$, at $318^\circ$ abs.

$$\therefore \qquad \text{its pressure} = \frac{96 \times 318}{980 \times 144} = 0{\cdot}216 \text{ lb./in.}^2$$

$$\therefore \text{ the total pressure } = 1{\cdot}382 + 0{\cdot}216 = 1{\cdot}598 \text{ lbs./in.}^2$$

**2.** Find the volume of air which must be removed per minute from a condenser into which 0·125 lbs. of air enter per minute with the exhaust steam. The temperature is 40°C., the vacuum 27″, and the barometer 29·95″.

A vacuum of 27″ gives a total pressure $= 1{\cdot}47$ lbs./in.$^2$

The vapour pressure (from tables) $= 1{\cdot}06$ „

$\therefore$ the air pressure $= 0{\cdot}41$ „

$$\therefore \text{ the vol. of the air per lb. } = \frac{96 \times 313}{0{\cdot}41 \times 144} = 510 \text{ ft.}^3$$

$\therefore$ the vol. of air to be extracted from the condenser

$$= 510 \times 0{\cdot}125 = 63{\cdot}6 \text{ lbs. per minute.}$$

**3.** The steam space in a surface condenser has a capacity of 60 ft.$^3$, the pressure is 2 lbs./in.$^2$, and the temperature 50°C. Find the weight of air present. (Mech. Sc. Trip , 1916.)

**4.** An air pump for a condenser is to give a vacuum of 26″ of mercury with a discharge temperature of 45°C. ; it may be taken that the ratio of air to steam by weight in the exhaust is 0·066. Assuming no leakage, and neglecting slip and clearance, find the volume swept through by the piston of the air pump per unit volume of water discharged. Barometer 30″. The density of air may be taken as 0·08 lb. per standard cubic ft. (Mech. Sc. Trip., 1913.)

ENGINE TRIALS.

## 56. To find I.H.P. from Indicator Diagram.

$s$ = "spring number" of spring used in indicator.

Then 1″ height on diagram = $s$ lbs./in$^2$.

Measure length of diagram = $l$ ins.

„ area „ = $a$ in$^2$. (by planimeter)

$\therefore$ mean height $= h = \frac{a}{l}$ ins.

$\therefore$ „ pressure $= P = hs$ lbs/in$^2$.

Let $N$ = r.p.m.

$L$ = length of stroke (Feet)

$A$ = nett piston area (in$^2$.)

= area of piston − area of piston rod.

Then **I.H.P.** $= \frac{\mathbf{PLAN}}{\mathbf{33000}}$.

This is the I.H.P. of one end of one cylinder. Repeat the process for both ends of all the cylinders, and add up the results. This gives I.H.P. of engine.

## 57. Heat Supplied to Engine per lb. of steam $= I_{w_1} + q_1L_1 - I_{w_2}$.

Where suffix (1) refers to steam as supplied,

and suffix (2) refers to feed water.

If $q_1$ be not known take it = 1.

In a condensing engine the steam used is the "*air-pump discharge*," which is the feed water.

## 58. The Heat Rejected per minute

= wt. of condensing water used per minute

× its rise of temperature.

This does not allow for losses such as radiation and leakage.

59. The **B.H.P.** $= \dfrac{2\pi rN(T_1 - T_2)}{33000}$,

where $r$ = radius of brake-wheel (Ft.)

$T_1 - T_2$ = the difference in the tensions (lbs.) at the two ends of the brake.

## Efficiency.

The Thermal efficiency $= \dfrac{\text{I.H.P.}}{\text{Heat supplied}}$.

The Mechanical efficiency $= \dfrac{\text{B.H.P.}}{\text{I.H.P.}}$.

60. A "Heat Balance Sheet" should be drawn up thus:—

| | |
|---|---|
| Heat supplied......... | Work done (i.h.p. expressed in Th. Units) ... ... ... |
| | Heat rejected, to condensing water, and other sources of loss according to data of problem, or details of trial ... |
| | Heat unaccounted for... ... |

The "heat unaccounted for" is calculated from the other items so that the right hand side adds up to the same as the left.

## EXAMPLES.

**1.** The following data are from a test of a steam engine using dry saturated steam, and fitted with a surface condenser:

I.H.P. = 560.
Air pump discharge per minute, 130·7 lbs.
Initial pressure, 210 lbs. per sq. inch abs.
Condenser pressure, 2·3 lbs. per sq. inch abs.
Circulating water per minute, 2266 lbs.
Initial temperature of circulating water, 15°C.
Final „ „ „ „ 43°C.

Find the thermal efficiency of the engine, and the percentage of heat unaccounted for. (Intercoll. Exam. Cambridge, 1911.)

The steam used per minute = 130·7 lbs.

At 210 lbs. per sq. in. $I_s = 670{\cdot}3$. lb. cals.

(i.) ∴ Heat supplied per minute

$$= 130{\cdot}7 \times 670{\cdot}3 = 87{,}500 \text{ lb. cals.}$$

At 2·3 lbs. per sq. in. $I_w = 55{\cdot}4$.

(ii.) ∴ Heat in condensed steam

$$= 130{\cdot}7 \times 55{\cdot}4 = 7230 \text{ lb. cals.}$$

(iii.) The heat carried away by the condensing water

$$= 2266\,(43 - 15) = 2{,}266 \times 28 = 63{,}300 \text{ lb. cals.}$$

(iv.) The indicated work done per minute

$$= \frac{560 \times 33{,}000}{1400} = 13{,}200 \text{ lb. cals.}$$

From (i.) and (ii.) the nett heat supply is

$$87{,}500 - 7230 = 80{,}270 \text{ lb. cals.}$$

Hence, from (iv.) the thermal efficiency is

$$\frac{13{,}200}{80{,}270} \times 100 = 16{\cdot}46\%.$$

Adding together (ii.) (iii.) and (iv.) the total heat accounted for is 83,730 lb. cals.

Therefore the heat unaccounted for is 3770 lb. cals. or 4·31% of the total gross supply.

**2.** The following data are furnished from a trial of a compound, double-acting, non-condensing engine :—

Cylinder diameters, 5″ and 10″. Stroke, 12″.
Brake wheel diameter, 5′. Nett load, 300 lbs.
Mean effective pressures: H.P. cylinder, 60 lbs./in.$^2$ and 56 lbs./in.$^2$ L.P. cylinder, 21 lbs./in.$^2$ and 18 lbs./in.$^2$
R.P.M. = 150.
Feed water, 500 lbs. per hour.

Determine the I.H.P., B.H.P., the mechanical efficiency, the steam used per I.H.P.-hour, and the indicated work per pound of steam. (Intercoll. Exam. Cambridge, 1908.)

(i.) The I.H.P.

Area of H.P. piston $= \pi \times 6{\cdot}25 = 19{\cdot}6$ in.$^2$

„ L.P. „ $= \pi \times 25 = 78{\cdot}5$ in.$^2$

Stroke = 1 ft.

Total indicated work per rev.

$$= [(60 + 56)\ 19{\cdot}6 + (21 + 18)78{\cdot}5]\,.\ 1.$$
$$= 5330 \text{ ft. lbs.}$$

$$\text{I.H.P.} = \frac{5330 \times 150}{33{,}000} = 24{\cdot}2$$

(ii.) The B.H.P. $= \dfrac{2\pi \times 2{\cdot}5 \times 150 \times 300}{33{,}000} = 21{\cdot}4$

(iii.) The mechanical efficiency $= \dfrac{21{\cdot}4}{24{\cdot}2} \times 100 = 88{\cdot}5\%$

(iv.) Steam used = 500 lbs. per hour.

$$= \frac{500}{24{\cdot}2} = 20{\cdot}7 \text{ lbs. per I.H.P. hour.}$$

(v.) Indicated work = 5330 × 150 ft. lbs. per minute.

Steam used per minute $= \dfrac{500}{60} = 8{\cdot}33$ lbs.

the indicated work done per lb. of steam

$$= \frac{5330 \times 150}{8 \cdot 33}$$

$$= 96{,}000 \text{ ft. lbs.}$$

**3.** In the test of a certain double-acting steam engine the indicator diagram had an area of 2·23 sq. ins., with a "60" spring. The length of the indicator diagram was 2·54″. The diameter of the piston is 15″, and the stroke 21″. Neglecting the area of the piston rod, find the I.H.P. of the engine at 200 R.P.M.

The mean height of the diagram $= \frac{2 \cdot 23}{2 \cdot 54} = 0 \cdot 916''$.

$\therefore$ the mean pressure $= 0 \cdot 916 \times 60 = 55$ lbs. per. sq. in.
Piston area $= \pi \times 7 \cdot 5^2 = 177$ in.$^2$, stroke $= 1 \cdot 75$ ft.

$$\text{The I.H.P.} = \frac{55 \times 1 \cdot 75 \times 177 \times 200}{33{,}000} \times 2$$

$$= 20 \cdot 7.$$

**4.** An engine works on the Rankine cycle between the limits of 150 lbs. per sq. in. abs., and 3 lbs./in.$^2$ abs. Compare the efficiency of this engine with that of the same engine when the steam is superheated to such a temperature that it is still just dry at the end of adiabatic expansion. Assume that the specific heat of superheated steam is 0·5. (Trin Coll. Cambridge, 1914.)

From the tables we take the following data:

| | $I_s$ | $I_w$ | $\phi_s$ | $\phi_w$ | $t$ |
|---|---|---|---|---|---|
| at 150 lbs./in.$^2$ | 666·7 | 183·9 | 1·578 | 0·515 | 181·2°C. |
| at 3 lbs./in.$^2$ | 622·9 | 61 | 1·885 | 0·202 | 60·9°C. |

(i.) Efficiency in the first case.

To find the dryness at the end of expansion, we have

$$1 \cdot 885\, q + (1 - q)\, 0 \cdot 202 = 1 \cdot 578$$

whence $q = 0 \cdot 815$

Then $I_2 = 0 \cdot 815 \times 622 \cdot 9 + 0 \cdot 185 \times 61 = 507$

and the efficiency $= \frac{666 \cdot 7 - 507}{666 \cdot 7 - 61} = 26 \cdot 4\%$.

(ii.) Let $t^1$ = the temperature, °C., to which the steam is superheated. Then

$$\phi_s{}^1 = 1{\cdot}578 + 0{\cdot}5 \log_e \frac{273 + t^1}{454{\cdot}2}.$$

If the steam is just dry at the end of expansion, we must have

$$1{\cdot}578 + 0{\cdot}5 \log_e \frac{273 + t^1}{454{\cdot}2} = 1{\cdot}885$$

or
$$\log_e \frac{273 + t^1}{454{\cdot}2} = 0{\cdot}614$$

which gives
$$t^1 = 566°\text{C.}$$

Then $I_1 = I_s{}^1 = 666{\cdot}7 + 0{\cdot}5\,(566 - 181{\cdot}2) = 869$

$I_2 = 622{\cdot}9$

and the efficiency $= \dfrac{869 - 622{\cdot}9}{869 - 61} = 30{\cdot}4\%$.

The ratio of the efficiencies $= \dfrac{30{\cdot}4}{26{\cdot}4} = 1{\cdot}15$.

**5.** A Rankine engine takes steam at 100 lbs./in.$^2$ abs. and exhausts at 16 lbs./in.$^2$ abs. Find its consumption in lbs. of steam per I.H.P. per hour, and the heat used per I.H.P. per minute. Find also these quantities when the pressure of the exhaust is 5″ of mercury. (Intercoll. Exam. Cambridge, 1909.)

**6.** The law of expansion in a certain steam engine being $pv^{1{\cdot}0646}$ = const., the steam being cut off at $\frac{1}{4}$ stroke, the initial pressure being 100 lbs./in.$^2$ the back pressure 3 lb./in.$^2$ and the clearance at one end being 0·2 of the volume swept out by the piston, compute the following: (*a*) The work done in half a revolution of the crankshaft, if the diameter of the piston is 12″, and the length of crank 12″. (*b*) The weight of steam used per stroke. (*c*) The work done per pound of steam. (R.N.C., Greenwich, 1908.)

**7.** Find the work done per pound of steam in a Rankine engine when the pressure of the steam at entry is 150 lbs./in.$^2$

abs., the dryness 0·98, and the exhaust pressure is 2 lbs./in.$^2$ abs.

If the steam is throttled until it is just dry at entry, find the change in the work done per pound of steam. (Mech. Sc. Trip., 1913.)

**8.** Find the H.P. of an engine, the details of the test being: Stroke 2′; piston diameter 12″; piston rod $2\frac{1}{2}$″; R.P.M. = 60. The card was the same for each end of the cylinder; it had an area of 2·3 sq. ins. with an "80" spring, the length between perpendiculars being 2·7″. (Special Exam. Cambridge, 1913.)

**9.** An engine working at 500 I.H.P. takes 15·4 lbs. of dry steam per H.P. hour at 215 lbs./in.$^2$ abs. The condenser temperature is 37°C. The circulating water is 4900 lbs. per minute and it rises from 15°C. to 29°C. Find the thermal efficiency and the heat unaccounted for. (Special Exam. Cambridge, 1913.)

**10.** The following observations were made upon a steam engine using dry saturated steam and fitted with a surface condenser:

Air-pump discharge per minute = 95 lbs.
Initial pressure = 180 lbs./in.$^2$
Condenser pressure = 1·5 lbs./in.$^2$
Circulating water used per minute = 1750 lbs.
I.H.P. = 330.

Find the thermal efficiency of the engine and determine the rise in temperature of the circulating water if 6% of the heat supply to the engine is allowed for leakage and radiation. (Intercoll. Exam. Cambridge, 1914.)

## ORIFICES AND NOZZLES.

61. **Theory of Steam Jet.** Let (Fig. 17) $v$ = velocity, $V$ = volume. Suppose the flow *adiabatic.*

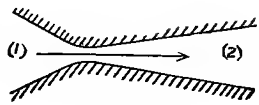


Fig. 17.

Increase of kinetic energy, 1 to 2,

$$= \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \text{ per lb.}$$

Work done on steam entering (1) by the steam behind it $= P_1V_1$.

Work done by steam leaving (2) on steam in front of it $= P_2V_2$.

Loss of internal energy $= E_1 - E_2$. Hence

$$\frac{v_2^2 - v_1^2}{2g} = E_1 - E_2 + P_1V_1 - P_2V_2,$$

or

$$\frac{\mathbf{v_2^2 - v_1^2}}{\mathbf{2g}} = \mathbf{I_1 - I_2} \qquad (1)$$

If $v_1$ be negligible or 0,

$$\frac{\mathbf{v_2^2}}{\mathbf{2g}} = \mathbf{I_1 - I_2} \qquad (2)$$

The heat drop, $I_1 - I_2$, is most easily found from Mollier's $\phi$ – I diagram —— draw a vertical line from the starting point on the $P_1$ line to the $P_2$ line.

By §49, $$I_1 - I_2 = \int_{P_2}^{P_1} VdP,$$

taken along the adiabatic.

$$\therefore \quad \frac{v_2^2 - v_1^2}{2g} = \int_{P_2}^{P_1} V.dP \qquad (3)$$

Suppose $PV^n = C$. Then

$$\frac{v_2^2 - v_1^2}{2g} = \frac{n}{n-1}(P_1V_1 - P_2V_2) = \frac{n}{n-1}\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}P_1V_1 \quad (4)$$

Hence, taking $v_1 = 0$, the velocity of discharge is given by

$$\frac{\mathbf{v}^2}{\mathbf{2g}} = \mathbf{I}_1 - \mathbf{I}_2 = \int_{P_2}^{P_1} VdP = \frac{\mathbf{n}}{\mathbf{n}-\mathbf{1}}\left\{\mathbf{1} - \left(\frac{\mathbf{P}_2}{\mathbf{P}_1}\right)^{\frac{\mathbf{n}-\mathbf{1}}{\mathbf{n}}}\right\}\mathbf{P}_1\mathbf{V}_1.$$

62. **Mass of Discharge from Nozzle.** At the smallest section let the pressure volume be $P_0$ and $V_0$, then

$$\frac{V_0}{V_1} = \left(\frac{P_1}{P_0}\right)^{\frac{1}{n}} \quad \therefore \quad V_0 = \frac{V_1}{\left(\frac{P_0}{P_1}\right)^{\frac{1}{n}}}$$

Mass discharged per unit area of section, then

$$Q = \frac{v}{V_0} = \frac{v}{V_1}\left(\frac{P_0}{P_1}\right)^{\frac{1}{n}} = \sqrt{\frac{2gn}{n-1}\,\frac{P_1}{V_1}\left\{\left(\frac{P_0}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_0}{P_1}\right)^{\frac{n+1}{n}}\right\}}.$$

63. **To find the value of $\frac{\mathbf{P}_0}{\mathbf{P}_1}$ which gives the maximum discharge.**

Let $\frac{P_0}{P_1} = x$, then, for maximum, $\frac{dQ}{dx} = 0$.

$$Q = \sqrt{\frac{2gn}{n-1}\,\frac{P_1}{V_1}} \cdot \sqrt{x^{\frac{2}{n}} - x^{\frac{n+1}{n}}}$$

For maximum $\sqrt{x^{\frac{2}{n}} - x^{\frac{n+1}{n}}}$ must be maximum.

$$\therefore \quad \frac{1}{2}\left(\frac{2}{n}x^{\frac{2}{n}-1} - \frac{n+1}{n}x^{\frac{1}{n}}\right)\left(x^{\frac{2}{n}} - x^{\frac{n+1}{n}}\right)^{-\frac{1}{2}} = 0.$$

$$\therefore \quad 2x^{\frac{1}{n}-1} - (n+1) = 0,$$

$$\therefore \quad x = \left(\frac{n+1}{2}\right)^{\frac{n}{1-n}}.$$

Hence for maximum discharge

$$\frac{P_0}{P_1} = \left(\frac{n+1}{2}\right)^{\frac{n}{1-n}}.$$

Taking $n = 1{\cdot}135$, this gives $\frac{P_0}{P_1} = 0{\cdot}58$.

64. **Maximum Discharge.** Inserting this value of $P_0/P_1$ in the expression for $Q$, gives

$$Q = \sqrt{\frac{2gn}{n-1} \cdot \frac{P_1}{V_1} \left\{ \left(\frac{n+1}{2}\right)^{\frac{2}{1-n}} - \left(\frac{n+1}{2}\right)^{\frac{1+n}{1-n}} \right\}}$$

per unit area of section. If $n = 1{\cdot}135$ and $g = 32{\cdot}2$,

$$Q = 3{\cdot}6 \sqrt{\frac{P_1}{V_1}}\,.$$

65. If the flow is into a region where the pressure is less than given above (about $0{\cdot}58\ P_1$) the steam must expand further, thereby gaining velocity, provided the jet is allowed to take up its natural shape.

To design a nozzle for a given discharge the area of the throat can be found by the above formula (§ 65), and the area at the discharge end can be found from the formula of (§ 63) substituting $P_2$ for $P_0$, neglecting friction.

## EXAMPLES.

**1.** Dry saturated steam is to expand through a nozzle from 200 lbs./in.$^2$ abs. to 5 lbs./in.$^2$ abs. Assuming $n = 1{\cdot}135$, find the diameters of the nozzle at the throat and at the discharge end for a flow of 50 lbs. of steam per minute.

From the tables $V_1 = 2{\cdot}316$ ft.$^3$/lb.

The flow per unit area

$$= 3{\cdot}6 \sqrt{\frac{200 \times 144}{2{\cdot}316}} = 402 \text{ lbs. per sec.}$$

$\therefore$ area required

$$= \frac{50}{60 \times 402} = 0{\cdot}002075 \text{ sq. ft.} = 0{\cdot}299 \text{ sq. ins.}$$

$\therefore$ diameter of throat $= 0{\cdot}618''$.

At the discharge end, the flow per unit area

$$=\sqrt{\frac{64\cdot4\times1\cdot135}{0\cdot135}\cdot\frac{200\times144}{2\cdot316}\cdot\left\{\left(\frac{5}{200}\right)^{\frac{2}{1\cdot135}}-\left(\frac{5}{200}\right)^{\frac{2\cdot135}{1\cdot135}}\right\}}$$

$$=\sqrt{541\times12430\left\{(0\cdot025)^{1\cdot762}-(0\cdot025)^{1\cdot88}\right\}}$$

$= 58\cdot8$ lbs. per sec.

$\therefore$ area required $= \dfrac{50\times144}{60\times58\cdot8} = 2\cdot04$ sq. ins.

$\therefore$ diameter must $= 1\cdot613''$.

**2.** Taking the same data as above, find the diameter at exit, assuming that 10% of the heat drop is lost in friction in the nozzle.

If the flow were frictionless the velocity of discharge would be given by

$$v^2 = 2g\ (I_1 - I_2)$$

actually it will be given by

$$v^2 = 0\cdot9 \times 2g\ (I_1 - I_2)$$

we find in the usual way $q_2 = 0\cdot82$, $I_1 - I_2 = 142$ Th. Units

$$v^2 = 0\cdot9\times64\cdot4\times142\times1400 = 11{,}500{,}000$$

$$v = 3400 \text{ ft./sec.}$$

The volume of the steam

$= 0\cdot82\times73\cdot39 = 60\cdot2$ ft.$^3$ per lb.

$\therefore$ discharge per unit area $= \dfrac{3400}{60\cdot2} = 5\cdot65$ lbs. per sec.

$\therefore$ area required $= \dfrac{50\times144}{60\times56\cdot5} = 2\cdot12$ in.$^2$

$\therefore$ diameter required $= 1\cdot643''$.

**3.** Find the principal dimensions of a nozzle to expand steam from 180 lbs./in.$^2$ abs. to 2 lbs./in.$^2$ abs., with a flow of 75 lbs. per minute, (*a*) neglecting friction, (*b*) assuming that 12% of the kinetic energy is lost in friction.

**4.** Steam at 200 lbs./in.$^2$ abs., superheated to 350°C., expands through a nozzle to 5 lbs./in.$^2$ Design a nozzle to deliver 60 lbs. of steam per minute, assuming adiabatic flow, and that 10% of the heat drop is lost in the nozzle.

THEORY OF INJECTORS.

66. Let suffix (1) refer to the steam as supplied to injector, suffix (2) refer to water entering injector, (3) refer to discharge.

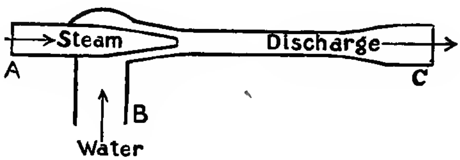



Fig. 18.

**To find the weight of water lifted per lb. of steam.** Let each pound of steam entering A draw $x$ lbs of water from B, so that $1 + x$ lbs. are delivered at C.

| | | |
|---|---|---|
| Heat energy per lb. of steam at A | $= q_1L_1 + I_{w1}$ | Th. Units. |
| Heat „ „ „ water entering | $= I_{w2}$ | „ |
| Kinetic „ „ „ „ „ | $= \dfrac{V_2^2}{2g}$ | Ft.-lbs. |
| Heat „ „ „ „ delivered | $= I_{w3}$ | Th. Units. |
| Kinetic „ „ „ „ „ | $= \dfrac{V_3^2}{2g}$ | Ft.-lbs. |

Where $V_2$ = velocity of water where it mixes with the steam, and $V_3$ = ditto in smallest section of delivery pipe. Then, by conservation of energy,

$$J(q_1L_1 + I_{w1}) + x\left(J \,.\, I_{w2} + \frac{V_2^2}{2g}\right) = (1 + x)\left(J \,.\, I_{w3} + \frac{V_3^2}{2g}\right) \quad (1)$$

The $V_2^2$ and $V_3^2$ terms can generally be neglected, then:

$$\mathbf{I_1 + x\, I_{w2} = (1 + x)\, I_{w3}} \quad \text{......................(1)}$$

Hence $x$ can be found.

Let $V_1$ = velocity of issuing steam. Then the conservation of momentum gives

$$\frac{V_1}{g} + x\frac{V_2}{g} = (1 + x)\frac{V_3}{g},$$

or $\mathbf{V_1 + x\, V_2 = (1 + x)\, V_3}$ .......................(2)

Equations (1) (in either form) and (2) are the equations to be used for solving injector problems.

If the injector lifts the water a term must be introduced into right hand side of (1) for the work done in lifting the water.

## EXAMPLES.

**1.** An injector is working on a boiler with 100 lbs./in$^2$. pressure: the feed water enters at 10°C. and is delivered to the boiler at 80°C. Find the weight of feed water supplied per lb. of boiler steam, assumed dry. (Mech. Sc. Trip., 1916.)

Let $x$ = the weight of feed water per lb. of steam.

The heat in the feed water = 10 Th. Units per lb.

" " steam supplied = 662 Th. Units per lb.

" " water when it enters the boiler = 80·3 Th. Units per lb.

Neglecting the work done we have

$$10x + 662 = (1 + x)\ 80{\cdot}3$$

whence $$x = \frac{661}{70{\cdot}3} = 9{\cdot}4 \text{ lbs.}$$

**2.** A steam ejector fitted to a destroyer will discharge 40 tons of water per hour, from the bilge, through a lift of 12 ft. The ejector is supplied with steam of a dryness fraction 0·9 at 250 lbs./in.$^2$ by gauge. The temperature of the bilge water is 10°C., and that of the water discharged from the ejector 29°C. Estimate approximately the quantity of steam used per ton of water pumped from the bilge, and the efficiency of this ejector as a pump. (R.N.C., Greenwich, 1912.)

**67. Classification of Turbines.** The principle of all steam turbines is the conversion of the thermal energy of the steam into kinetic energy, by letting the steam discharge, through nozzles, against blades or vanes fixed to rotating wheels. Turbines may be broadly divided into (1) *Impulse Turbines*, in which the total fall of pressure takes place in the nozzles, only the velocity changing in the passage through the blades; (2) *Reaction Turbines*, in which the pressure of the steam is reduced, as well as the velocity, by passing through the vanes.

TURBINE TYPES.

| Type | Name | Description |
|---|---|---|
| Single Stage. | De Laval. | Impulse. 1 set of fixed nozzles and 1 set of moving blades. |
| Multi-Stage. | Rateau<br>Zoelly | Each stage consists of a set of nozzles and 1 ring of blades mounted on a wheel fixed to shaft. Each is an impulse turbine; Rateau has 20 to 30 stages, Zoelly about 10. |
| | Parsons. | Called reaction, but really partly reaction and partly impulse. Each stage has 1 set of fixed blades and 1 set of moving blades. |
| | Curtis. | One or more stages, each consisting of one set of nozzles, followed by alternate rings of moving and fixed blades. |

**68. Velocity Diagram for Impulse Turbines.**

In Fig. 19, OA $= v_1 =$ velocity of steam leaving nozzle, and AB $= u =$ velocity of blades. Then OB $= v_2 =$ velocity of steam relative to blades, which gives the proper angle for the blades at entry. The steam comes out of the channel between the blades with the same relative velocity, DE $= v_3$. Then, if FD $= u$, FE $= v_4 =$ absolute velocity of steam at exit, which must be as small as possible, as the kinetic energy is wasted, and FE should be as nearly as possible perpendicular to FD.

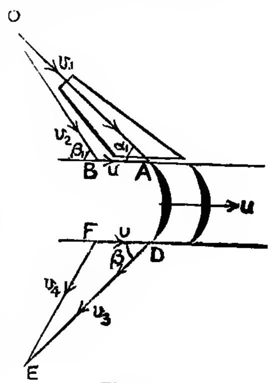



Fig. 19.

69. **The Velocity Diagram for a Parsons' Turbine** is shown in Fig. 20, for one stage constructed in

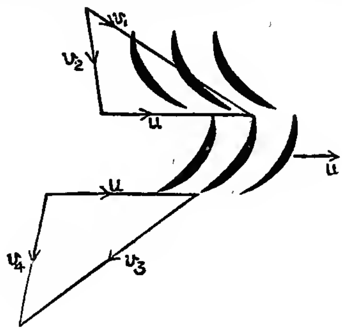



Fig. 20

the same way as above. In the diagram,

$u$ = velocity of moving blades.

$v_0$ = velocity of steam entering fixed blades.

$v_1$ = velocity of steam leaving fixed blades and entering moving blades.

$v_2$ = relative velocity of same.

$v_3$ = relative velocity of steam leaving moving blades to enter the next set of fixed blades.

$v_4$ = absolute velocity of same.

70. **Output of Single-Stage Impulse Turbines** (De Laval) (see Fig. 19).

The *work done* per lb. of steam = change of K.E.

$$= \frac{v_1^2 - v_4^2}{2g}.$$

The energy supplied $= \dfrac{v_1^2}{2g}$ per lb.

$\therefore$ The efficiency $= \dfrac{v_1^2 - v_4^2}{v_1^2}.$

If $\beta_2 = \beta_1$ (which is usual), Fig. 19 can be drawn as in Fig 21, since $v_3 = v_2$, where OK is drawn $= v_4 =$ OC, and at same angle as OC.

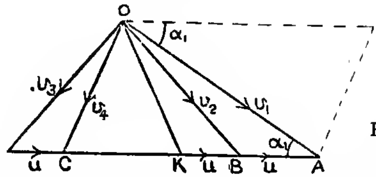



Fig. 21.

$$v_4^2 = OK^2 = OA^2 + AK^2 - 2\,OA\,.\,AK \cos\alpha_1$$
$$= v_1^2 + 4\,u^2 - 4\,u\,v_1 \cos\alpha_1,$$
$$\therefore v_1^2 - v_4^2 = 4\,u\,(v_1 \cos\alpha_1 - u)$$
$$\therefore \text{ efficiency} = \eta = \frac{v_1^2 - v_4^2}{v_1^2} = \frac{4u}{v_1}\left(\cos\alpha_1 - \frac{u}{v_1}\right).$$

For maximum efficiency $\quad \cos\alpha_1 = \dfrac{2u}{v_1}$,*

then max. efficiency $= \cos^2\alpha_1$.

The process to be followed when $\beta_1 \neq \beta_2$, and $v_3 \neq v_2$ will be seen from the example below.

71. **Output of Reaction (Parsons, etc.) Turbines** (Fig. 20).

For one stage:

Change of K.E. in fixed blades $= \dfrac{v_1^2 - v_0^2}{2g} = W_1$

" " moving " $= \dfrac{v_3^2 - v_2^2}{2g} = W_2$

K.E. carried away $= \dfrac{v_4^2}{2g}$,

$\therefore$ work done per lb. of steam is

$$W = \frac{v_1^2 - v_0^2}{2g} + \frac{v_3^2 - v_2^2}{2g} - \frac{v_4^2}{2g}$$

and $\qquad$ Efficiency $= \dfrac{W}{W_1 + W_2}$.

Hence, to find the efficiency, find the $v$'s, substitute their values in the expressions for $W$, $W_1$, and $W_2$, and hence find the efficiency.

* Differentiate $\eta$ with respect to $u$.

## EXAMPLES.

1. In a De Laval turbine, in which the full pressure drop is used up in the nozzles, and the blades have their outlet and inlet angles equal, steam is supplied dry and saturated at 180 lbs./in.$^2$, and the exhaust pressure is 2·5 lbs./in.$^2$ The peripheral speed of the blades is 1250 ft./sec. and the nozzles make an angle of 20° with the direction of motion of the blades. Assuming adiabatic flow in the nozzles and neglecting friction, find the velocity of discharge from the nozzles, the inlet and outlet angles of the blades, and the work done per pound of steam. (Mech. Sc. Trip., 1914.)

The velocity of discharge from the nozzles is given by (§ 61)

$$\frac{v_1^2}{2g} = I_1 - I_2.$$

$I_1 - I_2$ is found as for a Rankine engine (p. ) or from a Mollier diagram. We find $I_1 - I_2 = 158$ lbs. calories.

$$v_1^2 = 158 \times 1400 \times 64{\cdot}4 = 14{,}200{,}000$$

$$\therefore \quad v_1 = 3770 \text{ ft./sec.}$$

Let $\beta$ = the blade angles.

$$\operatorname{Tan} \beta = \frac{3770 \times \sin 20^\circ}{3770 \cos 20^\circ - 1250} = 0{\cdot}564$$

$$\therefore \quad \beta = 29{\cdot}4^\circ.$$

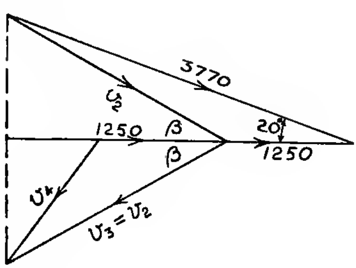



Fig. 22.

From the figure :—

$$v_3^2 = v_2^2 = 3770^2 + 1250^2 - 2 \,.\, 3770 \,.\, 1250 \,.\, \cos 20^\circ$$
$$= 6{,}910{,}000$$

$\therefore \quad v_3 = v_2 = 2630$ ft./sec.

Again : $v_4^2 = 1250^2 + v_3^2 - 2v_3 . 1250 . \cos \beta$

$= 2,740,000$

$\therefore \quad v_4 = 1655$ ft./sec.

The work done per lb. of steam

$$= \frac{v_1^2 - v_4^2}{2g} = 178,000 \text{ ft. lbs.}$$

(For max. efficiency we should have $\cos \alpha$

$= \frac{2500}{3770} = \cdot663$, or $\alpha = 48\cdot5°$ instead of $20°$.)

2. A De Laval turbine has blade angles of 30° at inlet and 38·5° at outlet. The nozzle axis makes 20° with the plane of the disc. The steam enters the blades without shock and leaves them with a relative speed which is 80% of the relative speed at entry. The steam is dry and saturated at entry and expands from 100 lbs./in.² to 1 lb./in.² absolute in the nozzle. Determine the thermal efficiency of the turbine, neglecting losses in the nozzle, the temperature of the boiler feed being 38°C. (Mech. Sc. Trip., 1912.)

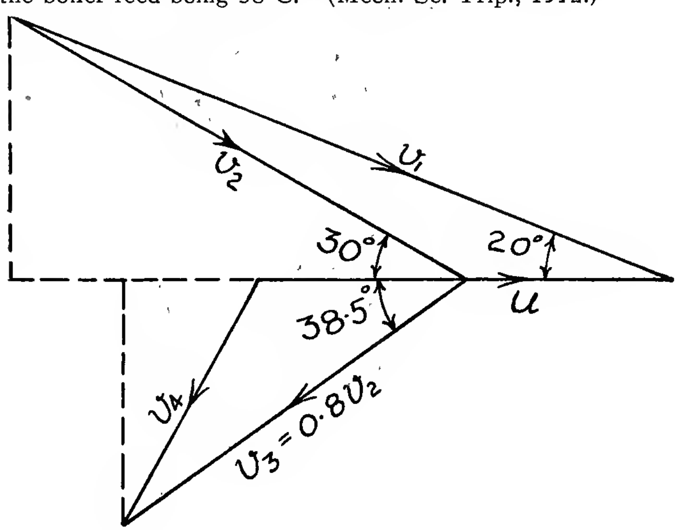



Fig. 23.

As before, $v_1 = \sqrt{2g\,(I_1 - I_2)\,J} = 3850$ ft./sec.

From the diagram :

$$v_2 = \frac{v_1 \sin 20^\circ}{\sin 30^\circ} = 2630 \text{ ft./sec.}$$

$$u = v_1 \cos 20^\circ - v_2 \cos 30^\circ = 1340 \text{ ft./sec.}$$

$$v_3 = 0{\cdot}8\ v_2 = 2100 \text{ ft./sec.}$$

$$v_4^2 = v_3^2 + u^2 - 2\,uv_3 \cos 38{\cdot}5^\circ = 1{,}800{,}000$$

$$\therefore \quad v_4 = 1340 \text{ ft./sec.}$$

The work done per lb. of steam

$$= \frac{v_1^2 - v_4^2}{2g} = 202{,}000 \text{ ft. lbs.}$$

$$= 144 \text{ lbs. calories.}$$

The thermal efficiency $= \dfrac{144}{I_1 - I_{w2}} = \dfrac{144}{624} = 0{\cdot}231.$

**3.** A Curtis (impulse) turbine has two stages, each with one set of nozzles, three rotating and two fixed sets of blades, as shown below diagrammatically. The nozzles are inclined

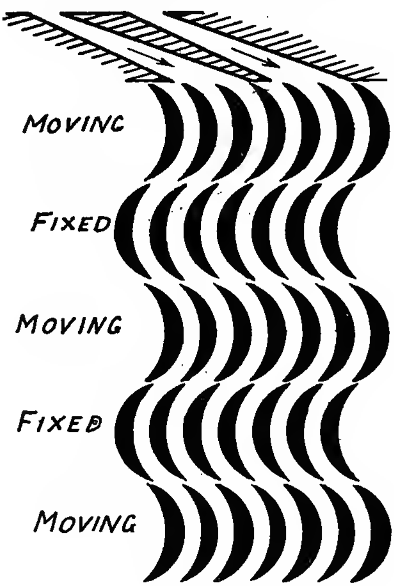



Fig. 24.

at 20° to the plane of rotation, and the peripheral velocity of the blades is 400 ft./sec. The moving blades all have the same angles at entry and exit; draw a velocity diagram for one stage and determine the angles of the moving and stationary blades. The velocity of the steam leaving the nozzles is 2800 ft./sec. Neglect friction losses.

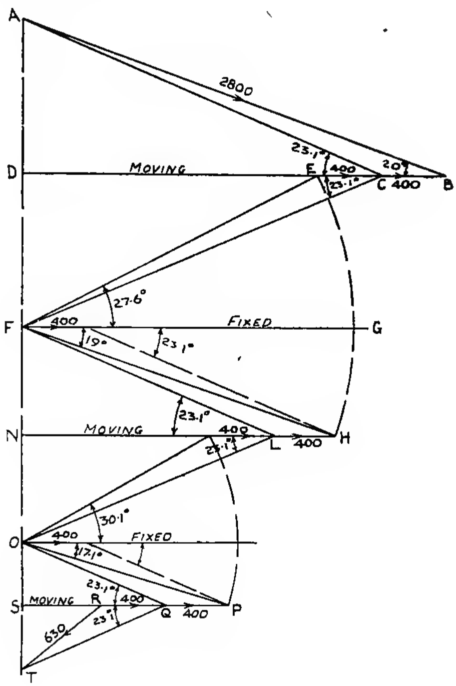



Fig. 25.

In the diagram, AB = velocity of steam leaving nozzle, and CB = velocity of blades. Then AC = relative velocity entering first row of moving blades, and ACD (= 23·1°) is the blade angle at entry and exit. This is to be the same for all the rows of moving blades.

Make EC = volocity of blades and CF = CA = relative velocity of steam. Then EF = absolute velocity of steam

leaving first moving blades and entering first fixed blades, so that the angle EFG ($= 27{\cdot}6^\circ$) is the entry angle of fixed blades.

Since the angle of entry for the second set of moving blades is to be $23{\cdot}1^\circ$, the direction of the relative velocity of the steam must make this angle with FG; make FK = velocity of blades, and draw KH at $23{\cdot}1^\circ$ to FG; then draw an arc with centre F radius FE cutting KH in H, so that FH = FE. Complete the parallelogram FKHL; then the angle GFH ($= 19^\circ$) gives exit angle for these fixed blades. Repeat the process until the last set of moving blades has been dealt with. MO and RT represent the absolute velocity of the steam leaving the second and third rows of moving blades. The angles of entry and exit for the second row of fixed blades are $30{\cdot}1^\circ$ and $17{\cdot}1^\circ$ respectively.

**4.** In a single stage impulse turbine compare the efficiency found by formula, neglecting friction, with the efficiency actually obtained under the following provisions:—

Velocity of exit from nozzles = 1800 ft./sec.
,, ,, wheel periphery = 600 ,,
Angle at which the inlet and outlet of the vanes are inclined to the direction of motion $= 30^\circ$.

Assume a loss of 8% due to friction in the moving vanes. (R.N.C. Greenwich, 1910.)

**5.** A 500 *kW* steam turbine using dry steam at a pressure of 140 lbs./in.$^2$ abs. consumes 22·6 lbs. of steam per kilowatt hour and condenses at a pressure of 1 lb. per. sq. in. abs. The condensing water measures 5620 lbs. per minute and its rise of temperature is $18{\cdot}3^\circ$C. Determine the dryness of the steam as it leaves the turbine and find the loss of work in the cylinder of the turbine due to the expansion not following the adiabatic line on the entropy-temperature diagram. (Mech. Sc. Trip., 1912.)

**6.** Draw to scale a velocity diagram for one complete constant pressure stage of a Curtis turbine, consisting of a set of nozzles, two sets of fixed and three sets of rotating blades; find the absolute speed of discharge and the speed of axial steam flow, from the last row of moving blades, having given that the blade velocity is 100 ft./sec., the speed of the steam leaving the nozzles 1200 ft./sec., and the angle of nozzles to the plane of rotation is 20°. (R.N.C. Greenwich, 1912.)

**7.** Steam issues from a nozzle, inclined at 17° to the circumference of a turbine wheel, at 3600 ft./sec. The peripheral speed of the blades is 1200 ft./sec. The inlet and outlet angles of the blades are equal. Find this angle, the velocity of discharge of the steam, the indicated H.P. per lb. of steam, and the efficiency of the blades neglecting all losses.

REFRIGERATION.

72. In the heat engine working direct we supply heat and obtain work; in the refrigerator we do work to extract heat from a substance which we wish to cool: we "lift" heat from one body to another of higher temperature at the expense of work.

73. $\frac{\text{Heat 'lifted' (extracted)}}{\text{work done}}$ is called the **coefficient of performance.**

74. Refrigerator working on reversed Joule cycle: **Bell-Coleman Refrigerator** (Fig 26).

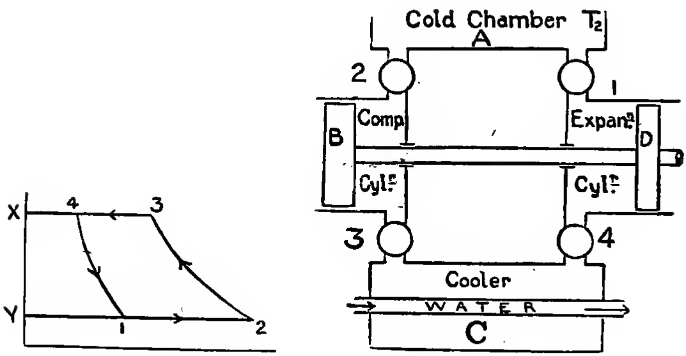



Fig. 26.

Used for cooling cold-chambers in ships. Air is drawn in by B from (Y2 in pv diagram) chamber A, and is compressed along 2 3 until the pressure equals the pressure in C, which raises the temperature above that of C. The compressed air is delivered (along 3X) into C. At the same time D draws (along X4) an equal quantity of air from C and expands it along 4 1, the temperature falling below that of A. The cold air is discharged into A on return stroke of D.

75. **To find the coefficient of Performance of the Bell-Coleman Refrigerator** (see Fig. 26).

Along 1 2, heat taken in from cold chamber

$$= k_p (T_2 - T_1) = Q_{12}$$

Along 3 4, heat given to cooler $= k_p (T_3 - T_4) = Q_{34}$

The work done = the area 1 2 3 4

$$= Q_{34} - Q_{12}$$

$$= k_p (T_3 - T_4 - \overline{T_2 - T_1});$$

$\therefore$ the coefficient of performance

$$= \frac{\text{The heat extracted}}{\text{work supplied}}$$

$$= \frac{k_p\ (T_2 - T_1)}{k_p\ (T_3 - T_4 - T_2 + T_1)}$$

$$= \frac{1}{\dfrac{T_3 - T_4}{T_2 - T_1} - 1}$$

$$= \frac{1}{\dfrac{T_4}{T_1} - 1} \qquad \left[\frac{T_3}{T_2} = \frac{T_4}{T_1}\right.$$

$$= \frac{T_1}{T_4 - T_1} = \frac{T_2}{T_3 - T_2}.$$

76. **Vapour Compression Refrigerators.** The chief substances used are $NH_3$, $CO_2$, $SO_2$. In the forward

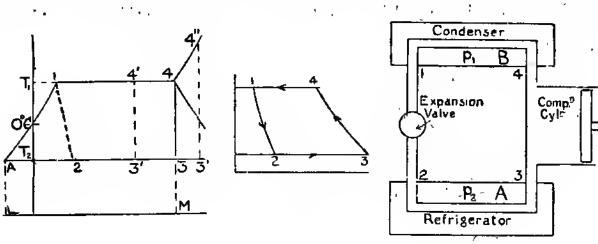



Fig. 27.

stroke of the piston, vapour is drawn from A (2 3), it is then compressed (3 4) and driven (4 1) into the condenser. An equal quantity of substance expands through the expansion

valve from B to A. This operation is irreversible. The $\phi - T$ diagram is shown to the left of Fig. 27, the compression ending with the vapour just dry, but the compression line might equally well be $3' \ 4'$ or $3'' \ 4''$.

1—2 is adiabatic passage through the valve,

3—4 is adiabatic compression in the cylinder.

## 77. **Equations for Vapour Refrigerators.**

From the $\phi$—T diagram:

Heat taken up $= q_3L_2 - q_2L_2 = (q_3 - q_2)L_2$

Work done in compressor $= I_4 - I_3$.

$$\therefore \quad \text{the coefficient of performance} = \frac{(q_3 - q_2)\,L_2}{I_4 - I_3}.$$

To find $q_2$ and $q_3$:—

Along 1—2 (see §47) we have

$$I_1 = I_2,$$

also $\quad I_2 = I_A + q_2L_2$

$$\therefore \; q_2L_2 = I_2 - I_A = I_1 - I_A \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i.)}$$

also $\quad q_3L_2 = \text{area L A 3 M} = (\phi_3 - \phi_A)T_2$

$$\text{or} \qquad q_3L_2 = (\phi_4 - \phi_A)T_2 \ldots\ldots\ldots\ldots \text{(ii.)}$$

Find $q_2L_2$ and $q_3L_2$ by (i.) and (ii.) from the tables, and so calculate the coefficient of performance by the formula above.

## 78. **To find the necessary volume of the cylinder.**

*Example:* For ammonia, when $T_1 = 20°C$, $T_2 = -20°C$, we find the heat lifted per pound of ammonia

$$= (q_3 - q_2)L_2 = 256 \quad \text{Th. Units.}$$

To lift 1000 Th. Units requires $\dfrac{1000}{256} = 3{\cdot}9$ lbs.

From the tables, the volume of 1 lb. at $-20°C$ is $10{\cdot}35$ ft$^3$. Hence volume required $= 3{\cdot}9 \times 10{\cdot}35$

$= \text{about } 40$ ft$^3$.

Thus the volume of the cylinder must be 40 ft$^3$. per 1000 Th. Units lifted.

## EXAMPLES.

**1.** In an ammonia refrigerating machine the lowest temperature is $-10°$C., and the liquid passes to the expansion valve at 30°C. If the compression be adiabatic, find the coefficient of performance if the vapour be just dry (*a*) at the end, (*b*) at the beginning of the compression.

(*a*) Following the method of § 77, from equation (i.) :—

$$q_2 L_2 = 322{\cdot}3\ q_2 = 28{\cdot}94 + 8{\cdot}02 = 36{\cdot}96$$

$$\therefore \quad q_2 = 0{\cdot}115$$

and from (ii.)

$$q_3 L_2 = 322{\cdot}3\ q_3 = (1{\cdot}055 + 0{\cdot}033)\ 263 = 286.$$

$$\therefore \quad q_3 = 0{\cdot}888$$

Then $I_4 = 318{\cdot}6$

$$I_3 = 0{\cdot}888 \times 314{\cdot}3 - 0{\cdot}112 \times 8{\cdot}02 = 289.$$

$$\text{The coeff. of performance} = \frac{286 - 36{\cdot}96}{318{\cdot}6 - 289} = 8{\cdot}9.$$

(*b*) In this case $q_3$ will be unity.

$$q_3 L_2 = 322{\cdot}3$$

$$I_3 = 314{\cdot}3$$

$$I_4 = 318{\cdot}6 + 0{\cdot}508\ (T^1 - 303)$$

where $T^1$ is the temperature reached at the end of compression, the sp. ht. of the superheated vapour being 0·508. Since the entropy is constant we have

$$\phi_4 = 1{\cdot}055 + 0{\cdot}508 \log_e \frac{T^1}{303} = \phi_3 = 1{\cdot}193$$

whence $T^1 = 398$

$$\therefore\ I_4 = 318{\cdot}6 + {\cdot}508 \times 95 = 366{\cdot}8.$$

$$\text{The coeff. of performance} = \frac{322{\cdot}3 - 36{\cdot}96}{366{\cdot}8 - 314{\cdot}3} = 5{\cdot}44.$$

**2.** In the above example find the cylinder volume required per 1000 Th. Units, lifted in each case.

**3.** A small ammonia refrigerator is found to convert 420 lbs. of water at 15°C. to ice at 0°C. in a 5 hours' run.

The I.H.P. of the compressor is 1·52 and the extreme temperatures of the ammonia are $-10°$C. and 25°C. Compare the actual coefficient of performance with the ideal. The latent heat of ice is 80 lbs. calories. (Mech. Sc. Trip., 1912.)

**4.** An ammonia machine works on the reversed Rankine cycle between the limits 20°C. and $-$ 10°C., the vapour being just dry and saturated at the end of compression. Calculate the percentage loss in the coefficient of performance owing to the absence of an expansion cylinder, if the liquid returns to the working cylinder through a regulating valve. (Mech. Sc. Trip., 1916.)

**5.** What would be the theoretical coefficient of performance of a refrigerating machine working on the reversed Stirling cycle?

**6.** An ammonia refrigerating plant is used for cooling a sterilized liquid having a sp. ht. of 0·96. The following data represent mean values of the readings taken at ten minute intervals:—

| | |
|---|---|
| Inlet temperature of the liquid ... ... | 23·8°C. |
| Outlet ,, ,, ,, ... ... | 12·3°C. |
| Weight of liquid cooled per minute ... | 156·5 lbs. |
| Horse-power exerted on compressor shaft | 15·3 |
| Inlet temperature of cooling water at ammonia condenser... ... ... | 8·9°C. |
| Outlet ,, ,, ,, ,, ... | 20°C. |
| Cooling water per minute ... ... | 153 lbs. |
| Heat rejected to compressor jacket per minute ... ... ... ... | 83·5 Th. Units |
| Cooling effect on bare pipes per minute | 222 Th. Units |

Make out a balance sheet for this plant and determine the thermal units not accounted for, and the ratio of the refrigerating effect to the work done in producing it. (Mech. Sc. Trip.. 1910.)

**7.** Compare the coefficients of performance of three refrigerating machines using ammonia, sulphur dioxide, and carbon-dioxide, between the limits $-10^\circ$C. and $25^\circ$C. In each case the vapour is to be supposed dry and saturated at the end of compression. Also find the ratios of the volumes of the compression cylinders if the machines have the same refrigerating effect per stroke.

# AIR COMPRESSORS.

79. **Action.** Air is sucked in through A, compressed, and delivered through B to a storage chamber.

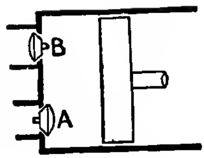



Fig. 28.

If the compression be slow it is isothermal; if rapid it is adiabatic.

Isothermal compression follows AC, and adiabatic follows AB, but in delivery the air generally falls to initial temperature. Hence, when the compression is adiabatic, work equal to ACB is wasted.

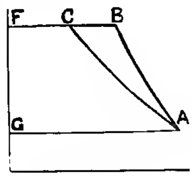



Fig 29.

The wasted work is reduced by dividing the compression into two or more stages and cooling between them.

80. **Single-Stage Air Compressor.** Neglecting clearance.

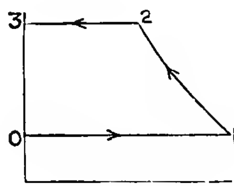



Fig. 30.

(i.) *Isothermal Compression.*

*To find the work done:*

Work done in compression $= p_1 v_1 \log_e \dfrac{p_2}{p_1} = w_1$

" " delivery $= p_2 v_2 = p_1 v_1 = w_2$

" *by* the air on the suction side of piston $= p_1 v_1 = w_3$

$W =$ Total work done $= w_1 + w_2 - w_3 = p_1 v_1 \log_e \dfrac{p_2}{p_1}$

(ii.) *If compression follow* pv$^n$ = *constant.*

$$W_1 = \frac{p_2 v_2 - p_1 v_1}{n - 1}, \qquad W_2 = p_2 v_2, \qquad W_3 = p_1 v_1,$$

$$\therefore \quad W = \frac{p_2 v_2 - p_1 v_1}{n - 1} + p_2 v_2 - p_1 v_1,$$

$$= \frac{n}{n-1} p_1 v_1 \left(\frac{p_2 v_2}{p_1 v_1} - 1\right),$$

**Work done**

$$= \frac{n}{n-1} RT \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \quad \text{.........................} \quad (1)$$

The **rise of temperature** is given by (see §19, ii.)

$$T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} T_1 \quad \text{.........................} (2)$$

The **heat taken up** by the air

$$= W_1 - k_v(T_2 - T_1)$$

$$= \frac{R(T_2 - T_1)}{n-1} - k_v(T_2 - T_1)$$

$$= \left(\frac{R}{n-1} - k_v\right)(T_2 - T_1) \text{.........................} \quad (3)$$

$$= \left(\frac{R}{n-1} - k_v\right) \frac{p_1 v_1}{R_1} \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}$$

81. **Two-stage Compressors.** Air is compressed along 1 2 to p$_2$, cooled to initial temperature, compressed along 3 4 and again cooled. Thus the area 2 3 4 4$'$ is saved.

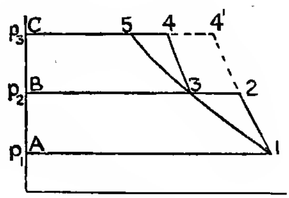



Fig. 31.

Area B 2 1 A $= \dfrac{n}{n-1} p_1 v_1 \left\{\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}$ by § 80.

Area C 4 3 B $\quad = \dfrac{n}{n-1}\, p_2 v_3 \left\{ \left(\dfrac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1 \right\}$ by § 80.

and $\quad p_2 v_3 = p_1 v_1.$

$$\therefore W = \text{B 2 1 A} + \text{C 4 3 B} = \frac{n}{n-1} R T_1 \left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2 \right\}.$$

## 82. To find the relation between the pressures so that W shall be a minimum.

$$\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} \quad \text{must be a minimum.}$$

Let $\quad x = p^{\frac{n-1}{n}}.$

Then $\quad y = \dfrac{x_2}{x_1} + \dfrac{x_3}{x_2}$ must be a minimum,

$$\frac{dy}{dx_2} = \frac{1}{x_1} - \frac{x_3}{x_2^2} = 0,$$

$$\therefore \quad x_2^2 = x_1 x_3$$

$\therefore \quad p_2^2 = p_1 p_3$ which is the relation sought.

## 83. To find the Cylinder ratios.

If the isothermal is $pv = C$, and the above condition is satisfied,

$$\frac{C^2}{v_2^2} = \frac{p_3}{p_1} \frac{C^2}{v_1^2},$$

$$\therefore \quad v_2 = v_1 \sqrt{\frac{p_1}{p_3}}.$$

If the stroke of both cylinders be the same, the ratio of the diameters is given by

$$d_2^2 = d_1^2 \sqrt{\frac{p_1}{p_3}},$$

$$\therefore \quad \frac{d_2}{d_1} = \sqrt[4]{\frac{p_1}{p_3}},$$

## 84. **Three-stage Compressors** (Fig. 32).

$W$ can be found as before, and we must have for minimum work

$$p_2^2 = p_1 p_3$$
$$p_3^2 = p_4 p_2$$
$$\therefore \quad \frac{p_4}{p_3} = \frac{p_3}{p_2} = \frac{p_2}{p_1},$$

and similarly for any number of stages; *i.e.* the pressures must be in G.P.

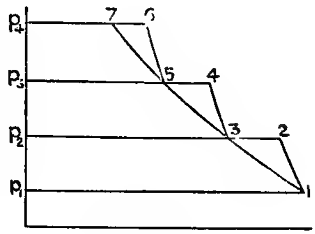


Fig. 32.

Similarly we can shew

$$\frac{d_2}{d_1} = \left(\frac{p_4}{p_1}\right)^{\frac{1}{6}} \qquad \frac{d_3}{d_1} = \left(\frac{P_4}{P_1}\right)^{\frac{1}{3}}.$$

## 85. **The effect of Clearance.**

In Fig. 33,

CL = stroke volume = $V$,

FL = clearance ,, $= cV$.

At the beginning of suction stroke, the air left behind expands, and the inlet valves do not open until $p_1$ is reached at H.

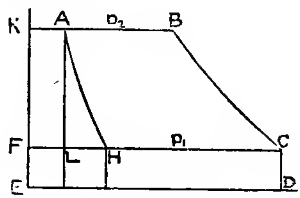


Fig. 33.

Since air does not accumulate in the cylinder,

$$\frac{\text{CH}}{\text{AB}} \text{ (Fig. 33)} = \frac{\text{CF}}{\text{AB}} \text{ (Fig. 34)}$$

(where there is no clearance), for AH and BC follow $pv^n = \text{const.}$

Then $\qquad v_1 = V + cV = (1 + c)V,$

and $\qquad v_2 = cV.$

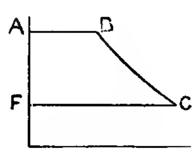


Fig. 34.

$\therefore$ the area KBCF

$$= \frac{n}{n-1} p_1 \,.\, V(1 + c)\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\},$$

also $$\text{FH} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} cV.$$

$$\therefore \text{ area KAHF} = \frac{n}{n-1} p_1 \,.\, cV \,.\, \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\},$$

$$\therefore W = \text{ABCH} = \frac{n}{n-1} p_1 V \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \left\{1 + c - c\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right\}.$$

And the volume sucked in per stroke

$$= \left\{1 + c - c\left(\frac{p_2}{p_1}\right)^{1/n}\right\} V,$$

instead of $V$.

## EXAMPLES.

**1.** An air compressor takes in air at a constant pressure $p_0$, and, after compressing it adiabatically, delivers it to a receiver at constant pressure $p_1$. Show that the work done by the compressor is given by $I_2 - I_1$, where $I_1$ and $I_2$ are the values of $I$ at the beginning and end of compression.

If the initial pressure $p_0$ is 15 lbs./in.$^2$, and the initial temperature is 15°C., find the temperature at which the air is delivered, the higher pressure $p_1$ being 500 lbs./in.$^2$ Find also the work done by the compressor per pound of air. (Intercoll. Exam. Cambridge, 1914.)

The proof of the first part of the question follows the lines of § 49.

The final temperature [§ 80,(2)] is

$$T_2 = \left(\frac{500}{15}\right)^{\frac{0\cdot4}{1\cdot4}} \times 288 = 785$$

$$or\ t_2 = 512°\text{C}.$$

The work done per pound of air [§ 80, (1)] is

$$\frac{1\cdot4}{0\cdot4} \times 96 \times 288 \left[\left(\frac{500}{15}\right)^{\frac{0\cdot4}{1\cdot4}} - 1\right]$$

$$= 167{,}000 \text{ ft. lbs.}$$

**2.** What is the ratio of the efficiencies of two air compressors, compressing air from 15 to 150 lbs./in.$^2$ abs., one in a single stage, the other in two stages of equal pressure ratios? The compression follows the law $pv^{1\cdot3} =$ const. The air is cooled to its initial temperature between the stages in the second case. Neglect clearance and mechanical losses. (Mech. Sc. Trip., 1913.)

In the single stage compressor the work done per lb. of air is

$$\frac{1\cdot3}{0\cdot3} \times 96 \times T_1 \left[10^{\frac{0\cdot3}{1\cdot3}} - 1\right] = 3\cdot04 \times 96 T_1.$$

If the compression were isothermal, no work would be wasted in heating the air, and the work done would be

$$96T_1 \log_e 10 = 2{\cdot}303 \times 96T_1.$$

Taking this as unity the efficiency of the single stage compressor $= \dfrac{2{\cdot}303}{3{\cdot}04} = 0{\cdot}758$, *or* $75{\cdot}8\%$.

For the two stage compressor we must have

$$\frac{p_1}{p_2} = \frac{p_2}{p_3}$$

$$\text{or } p_2 = \sqrt{p_1 p_3} = \sqrt{2250} = 47{\cdot}5.$$

The work done (§ 81) per pound of air

$$= \frac{1{\cdot}3}{0{\cdot}3} \times 96 \times T_1 \left[\left(\frac{p_2}{p_1}\right)^{\cdot 231} + \left(\frac{p_3}{p_2}\right)^{0{\cdot}231} - 2\right]$$

$$= 2{\cdot}64 \times 96\ T_1$$

The efficiency $= \dfrac{2{\cdot}303}{2{\cdot}64} = 0{\cdot}874$, *or* $87{\cdot}4\%$.

The ratio of the efficiences is

$$\frac{2 \text{ stage}}{1 \text{ stage}} = \frac{87{\cdot}4}{75{\cdot}8} = 1{\cdot}15.$$

**3.** A single stage compressor compresses air from 15 to 150 lbs./in.$^2$ abs., the clearance volume being 1 % of the piston displacement volume. At what point of the suction stroke will the inlet valve open if the spring on it has negligible tension? (Mech. Sc. Trip., 1912.)

Referring to Fig. 33, we have $C = 0{\cdot}01$.

$$\text{FH} = 10^{\frac{1}{1{\cdot}4}} \times 0{\cdot}01V = 0{\cdot}05176V$$

$$\text{FL} = 0{\cdot}01V$$

$$\therefore \quad \text{LH} = 0{\cdot}04176V$$

$$\text{LC} = V$$

The inlet valve will open at H, *i.e.* at $0{\cdot}04176$ of the stroke.

**4.** Calculate the H.P. required to drive a single stage compressor which takes in 250 ft.$^3$ of air per minute and

compresses it adiabatically from 15 lbs./in.$^2$ to 150 lbs./in.$^2$ The clearance is 0·5 % of the stroke volume, and the mechanical efficiency of the compressor 85 %.

**5.** A three stage compressor is used to charge a receiver of 20 ft.$^3$ capacity from atmospheric pressure to 2000 lbs./in.$^2$ Find the total work done while the pressure in the receiver rises to the final first and second stage pressures ($n = 1{\cdot}3$). Assume that the compressor has been designed for maximum efficiency when working against a steady back pressure of 2000 lbs./in.$^2$ (R.N.C. Greenwich, 1912.)

**6.** The clearance volume in a single stage compressor is 0·5 % of the volume swept out by the piston, which is 2 ft.$^3$ Calculate the number of strokes of the piston required to deliver 300 ft.$^3$ at 100 lbs./in.$^2$, the volume being measured at the higher pressure after cooling to the initial temperature of 15·5C°. (R.N.C. Greenwich, 1912.)

**7.** In a system of power transmission by compressed air, air at 15°C. is compressed adiabatically to 100 lbs./in.$^2$ abs. It passes to the motor through pipes in which the temperature falls to the original atmospheric temperature, the pressure remaining constant. The air then expands adiabatically in the motor cylinder, the expansion ratio being 3·5 : 1. Find the efficiency of the transmission neglecting friction losses. (Trin. Coll. Cambridge, 1913.)

COMBUSTION.

86. **Data:**

| | | | | | | |
|---|---|---|---|---|---|---|
| Element :— | H | O | N | C | S | |
| Atomic wt. :— | 1 | 16 | 14 | 12 | 32 | |
| Gas :— | $H_2O$ | $CO$ | $CO_2$ | $C_2H_4$ | $NH_3$ | $CH_4$ |
| Molecular wt. : — | 18 | 28 | 44 | 28 | 17 | 16 |

Heat given out when

1lb. of C burns in O to form $CO_2$ = 8060 Th. Units.
,, C ,, ,, O ,, $CO$ = 2450 ,,
,, H combines with O ,, $H_2O$ = 34500 ,,

The composition of air is $\frac{O}{N} = \frac{23}{77}$ by weight,

or $\frac{O}{Air} = \frac{23}{100}$ ,,

At 0° C, 760 mm. pressure, 1 cub. ft. of air = 0·0807 lbs.

87. **Fundamental Combinations.**

(i.) $$C + O_2 = CO_2$$

To obtain the proportions by weight in which C and O combine, substitute the atomic weights of the elements :—

$$12 + 32 = 44$$

*i.e.* 12 lbs. of C combine with 32 lbs. of O to give 44 lbs. of $CO_2$ or 1 lb. of C requires $\frac{8}{3}$ lbs. of oxygen for complete combustion.

Hence for complete combustion

$$1\text{lb. of C requires} \begin{cases} 2{\cdot}66 \text{ lbs. of oxygen,} \\ \text{or } 11{\cdot}5 \text{ lbs. of air} \\ = 142{\cdot}5 \text{ ft.}^3 \text{ at } 0° \text{ C. } 760 \text{ mm.} \\ = 150 \text{ ft.}^3 \text{ at } 15° \text{ C. } 760 \text{ min.} \end{cases}$$

(ii.) $$C + O = CO$$
$$12 \quad 16 \quad 28$$

Whence, as above, 1 lb. of C requires 5·77 lbs. of air.

(iii.) $$2H_2 + O_2 = 2H_2O$$
$$4 \quad 32 \quad 36$$
$$1 \quad 8 \quad 9$$

*i.e.* 1 lb. of hydrogen requires 8 lbs. of oxygen for complete combustion, and forms 9 lbs. of water, giving out 34,500 Th. Units.

## 88. **Draught in a Chimney.**

Let $v$ = velocity of gases; $h_a$ = height of column of air corresponding with difference of pressure (draught) above and below grate. Then

$$v = \sqrt{2gh_a}.$$

Let $h$ = draught in inches of water (1 inch of water is equal to 5·2 lbs. per ft.$^2$) Then

$$h_a = \frac{5{\cdot}2h}{0{\cdot}0807} = 64{\cdot}5h \text{ at standard temp. and pressure}$$

$$\therefore \qquad v = \sqrt{129gh} = \sqrt{4150h} \text{ ft./sec.}$$

Let $H$ = height of chimney above grate (feet)

$T_1$ = temperature inside chimney (abs.)

$T_2$ = ,, outside ,,

$A$ = area of section of chimney (ft.$^2$)

$m$ = lbs. af air per lb. of fuel.

Vol. of chimney = $AH$.

$\therefore$ wt. of this vol. of external air

$$= AH\,\frac{0{\cdot}0807 \times 273}{T_2}$$

$\therefore$ wt. of this vol. of chimney gases

$$= AH.\frac{0{\cdot}0807 \times 273}{T_1}.\frac{m+1}{m}.$$

Difference between these is equivalent to draught, and so

$$AH\left(\frac{1}{T_2} - \frac{m+1}{m}.\frac{1}{T_1}\right)0{\cdot}0807 \times 273 = 5{\cdot}2\,Ah$$

$$H\left(\frac{1}{T_2} - \frac{m+1}{m}\cdot\frac{1}{T_1}\right) = 0{\cdot}236h.$$

89. **Specific Heat of Gas Mixtures.** The sp. ht. of a mixture of $m_1$ lbs. of a gas of sp. ht. $k_1$, $m_2$ lbs. of a gas of sp. ht. $k_2$, etc., is

$$k = \frac{m_1 k_1 + m_2 k_2 + \ldots\ldots}{m_1 + m_2 + \ldots\ldots}$$

The following may be taken as the mean values of $k_p$ for the gases named :—

| | |
|---|---|
| $CO_2$ ...... | 0·216 |
| $CO$ | 0·245 |
| $O_2$ | 0·218 |
| $N_2$ | 0·244 |
| $H_2O$ ...... | 0·480 |

## EXAMPLES.

**1.**—A certain coal has the following composition by weight: C = 79·7; H = 4·9; O = 10·3, %; the rest is ash, nitrogen, etc. Find its gross and nett calorific values, and the air supply required per lb. of coal. (Mech. Sc. Trip., 1910.)

Consider 100 lb. of coal:

What O there is exists in the form of moisture.

∴, by §83 (ii.), there is $\frac{1}{8}$ as much H as O in combination with the O,

$$\therefore \quad \text{H present as } H_2O = \frac{10\cdot 3}{8} = 1\cdot 29 \text{ lbs.}$$

and the O ,, ,, $= 10\cdot 3$.

$\therefore$ total weight of $H_2O = 10\cdot 3 + 1\cdot 29 = 11\cdot 59$ lbs.,

and nett weight of free H $= 4\cdot 9 - 1\cdot 29 = 3\cdot 61$ lbs.

*To find the calorific value :—*

Heat produced by combination of 79·7 lbs. of C

$= 79\cdot 7 \times 8060 = 642{,}000$ Th.U.

ditto, 3·61 lbs. of H $= 3\cdot 61 \times 34{,}500 = \underline{124{,}400}$

$766{,}400$

$\therefore$ calorific value of 1 lb. = 7664 Th. Units.

This is called the **gross calorific value.** A lot of this heat goes up the chimney, but we charge this to the boiler, not the coal. The coal is charged with the latent heat of the steam carried away, which is subtracted from the gross calorific value to give the **nett calorific value:**—

Total wt. of $H_2O$ originally present = 11·59 lbs.

The combustion of the 3·61 lbs. of H with the oxygen of the air gives $3{\cdot}61 \times 9 = 32{\cdot}49$ lbs.

$\therefore$ total wt. of $H_2O$ going up the chimney = 44·08 lbs.

The latent heat of this $= 44{\cdot}08 \times 539 = 23700$ Th.U. or 237 per lb. of coal burnt.

$\therefore$ nett calorific value $= 7664 - 237 = 7427$ Th. Units per pound.

*To find the amount of air required for combustion* :—

By (i.) of §83, for 79·7 lbs. of carbon we require

$$79{\cdot}7 \times \tfrac{32}{12} = 212{\cdot}5 \text{ lbs. of O}$$

By (iii.) of §83, for 3·61 lbs. of H we require $3{\cdot}61 \times 8$
$= 28{\cdot}88$ lbs. of O.

$\therefore$ each 100 lbs. of coal requires $212{\cdot}5 + 28{\cdot}88$

$$= 241{\cdot}38 \text{ lbs. of O}$$
$$= 241{\cdot}38 \times \tfrac{100}{23} \text{ lbs. of air}$$
$$= 1049 \text{ lbs.,}$$

or, air req. per lb. of coal = 10·49 lbs.

**2.**—The flue gases from a boiler are found to contain, by volume, $CO_2 = 12\%$, $CO = 1{\cdot}3\%$, $O_2 = 6{\cdot}5\%$. Find the actual supply of air to the boiler. (Mech. Sc. Trip., 1910.)

The volumes present are $CO_2 : CO : O_2 = 12 : 1{\cdot}3 : 6{\cdot}5$

The relative densities are .............. $= 44 : 28 : 32$

$\therefore$ The weights present are

$$CO_2 : CO : O_2 = 12 \times 44 : 1{\cdot}3 \times 28 : 6{\cdot}5 \times 32$$
$$= 528 \quad : 36{\cdot}4 \quad : 208$$

1 lb. of $CO_2$ requires $\tfrac{3}{11}$ lbs. of C and $\tfrac{8}{11}$ lbs. of $O_2$,

$\therefore$ 528 „ „ 143·7 „ C „ 384 „ „ $O_2$;

1 lb. of CO requires $\tfrac{12}{28}$ lbs. of C and $\tfrac{16}{28}$ lbs. of $O_2$,

$\therefore$ 36·4 lbs. „ „ 15·6 „ C „ 20·8 „ „ $O_2$.

Hence we can say, that $143 \cdot 7 + 15 \cdot 6 = 169 \cdot 3$ lbs. of carbon were burnt, producing 528 lbs. of $CO_2$ and 36·4 lbs. of CO, and that $384 + 20 \cdot 8 = 404 \cdot 8$ lbs. of $O_2$ were supplied for combustion, and 208 lbs. besides, or 612·8 lbs. of $O_2$ altogether. Thus each lb. of C had

$$\frac{612 \cdot 8}{169 \cdot 3} = 3 \cdot 62 \text{ lbs. of } O_2 \text{ supplied,}$$

$$3 \cdot 62 \times \tfrac{100}{23}$$

$$= 15 \cdot 7 \text{ lbs. of air.}$$

**3.**—Find the specific heat, at constant pressure, of the flue gases whose composition by weight is

| | |
|---|---|
| $CO_2$...... | 17·5 |
| CO | 1·2 |
| $O_2$ | 6·9 |
| $N_2$ | 74·4 |
| | 100·0 |

The sp. ht. is

$$\frac{17 \cdot 5 \times 0 \cdot 216 + 1 \cdot 2 \times 0 \cdot 245 + 6 \cdot 9 \times 0 \cdot 218 + 74 \cdot 4 \times 0 \cdot 244}{100}$$

$= 0 \cdot 237$.

**4.** The gas from a suction gas plant gives the following analysis by volume :—

| | |
|---|---|
| $CH_4$...... | 0·65 |
| $CO_2$ | 6·57 |
| $H_2$ | 18·73 |
| $CO$ | 25·07 |
| $N_2$ | 48·98 |
| | 100·00 |

Find the volume of air required for the complete combustion of 100 ft.$^3$ of the gas, and the lower calorific value of the gas per standard cubic foot. (Mech. Sc. Trip., 1907.)

**5.** A sample of petrol consists of 80% of $C_6H_{14}$, 18% of $C_7H_{16}$ and 2% of $C_5H_{12}$. Find the weight of air required for the complete combustion of 1 lb. of this petrol and the lower calorific value. (Trin. Coll. Cambridge, 1912.)

**6.** A sample of coal contains 80% by weight of carbon and 5% of hydrogen, some of which is combined in the form of moisture. When the sample is tested the lower calorific value is found to be 7500 Th. Units per pound; find the percentage of moisture originally present. (Mech. Sc. Trip., 1916.)

**7.** The boiler room of a battleship contains six Yarrow boilers; the dimensions of each of the grates being $7' \times 8'6''$. Forty pounds of coal are burned per square foot of grate per hour, the composition being: C, 90%; H, 4%; O, 2%. The fan for the forced draught produces a pressure in the suction duct of 0·04″ of water below atmospheric pressure. If the supply of air by the fan is 100% in excess of the minimum required for complete combustion, calculate the section of the air ducks, the temperature of the air being 15°C., the specific volume at that temperature and atmospheric pressure being 13 ft.$^3$ per lb. (Mech. Sc. Trip., 1912.)

# INTERNAL COMBUSTION ENGINES.

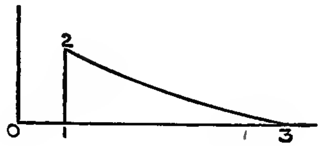



Fig. 35.

90. *Cycles.*

We shall work out the efficiencies of internal combustion engines, working on different cycles.

(1) Fig. 35. 0—1 is suction.

1—2 is explosion.

2—3 is adiabatic expansion.

The only supply of heat is along 1—2, and is instantaneous.

Heat supplied $= Q_1 = k_v (T_2 - T_1)$,

" rejected $= Q_2 = k_p (T_3 - T_1)$,

$$\therefore \quad \text{efficiency} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{k_p}{k_v} \cdot \frac{T_3 - T_1}{T_2 - T_1} = 1 - \gamma \frac{T_3 - T_1}{T_2 - T_1}.$$

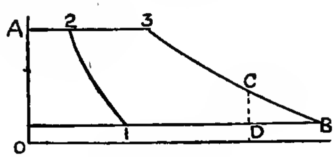



Fig. 36.

(2) Fig. 36. Two cylinders used, a compressing pump cylinder and a motor cylinder. The action is :—

0—1 = suction
1—2 = adiabatic compression } pump.

2—A = passage into receiver.

A23 = passage into motor cylinder, and supply of heat

3C B = adiabatic expansion.

With complete expansion, we have, since this amounts to a Joule cycle (see §36),

$$\text{the efficiency} \quad = 1 - \left(\frac{v_2}{v_1}\right)^{\gamma - 1}.$$

With *incomplete expansion*, assume cooling at constant $v$, as CD, and then at constant pressure, D1. Then

$$Q_2 = k_v (T_C - T_D) + k_p (T_D - T_1)$$

$$\text{and efficiency} = 1 - \frac{\frac{1}{\gamma}(T_C - T_D) + (T_D - T_1)}{T_3 - T_2}.$$

(3) **The Otto Cycle** (Fig. 37). The compression takes place in the motor cylinder

0—1 = suction (1st stroke) of explosive mixture.
1—2 = adiabatic compression (2nd stroke).
2—3 = explosion = heat supply at constant volume.
3—4 = adiabatic expansion (3rd stroke).
4—1 = cooling at constant volume.
1—0 = exhaust (4th stroke).

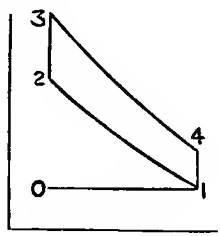


Fig. 37.

As in §37 we have,

$$\text{efficiency} \quad = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{v_2}{v_1}\right)^{\gamma - 1}.$$

91. **The Diesel Engine.** The distinctive feature is the adiabatic compression of air to such a high temperature that it ignites the liquid fuel injected into the cylinder (Fig. 38). In Fig. 38, the action is as follows:

0 1 = suction of air into the cylinder (1st stroke).

1 2 = adiabatic compression of air (2nd stroke).

$$\left.\begin{array}{l} 2\ 3 = \text{injection of fuel and burning} \\ 3\ 4 = \text{adiabatic expansion} \end{array}\right\}\ \text{(3rd stroke).}$$

4 1 = cooling at constant volume.

1 0 = exhaust (4th stroke).

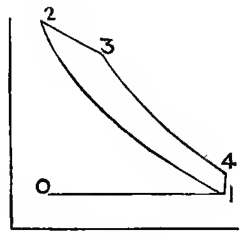



Fig. 38.

Assume that the burning (2—3) is isothermal.

To find the efficiency :

$$Q_1 = \text{heat supplied} = RT_2 \log_e \frac{v_3}{v_2}.$$

$$Q_2 = \text{heat rejected} = k_v (T_4 - T_1).$$

$$= k_v \left\{ T_3 \left( \frac{v_3}{v_4} \right)^{\gamma-1} - T_1 \right\}$$

$$= k_v \left\{ T_2 \left( \frac{v_3}{v_4} \right)^{\gamma-1} - T_1 \right\}$$

$$= k_v \left\{ T_1 \left( \frac{v_1}{v_2} \right)^{\gamma-1} \left( \frac{v_3}{v_4} \right)^{\gamma-1} - T_1 \right\}$$

$$= k_v\, T_1 \left\{ \left( \frac{v_3}{v_2} \right)^{\gamma-1} - 1 \right\},$$

$$\therefore \quad \text{efficiency} \quad = 1 - \frac{k_v T_1 \left\{ \left( \frac{v_3}{v_2} \right)^{\gamma-1} - 1 \right\}}{RT_2 \log_e \frac{v_3}{v_2}}.$$

If, instead of assuming that the burning along 2 3 is isothermal, we assume that it takes place at constant *pressure*, we shall have

$$Q_2 = k_v (T_4 - T_1)$$

$$Q_1 = k_p (T_3 - T_2)$$

and the efficiency

$$= 1 - \frac{k_v (T_4 - T_1)}{k_p (T_3 - T_2)}.$$

92. **Air Standard.** Since the working substance always consists mainly of air, it is usual to suppose that it is all air, and in the above expressions give $k_v$, $k_p$, and $\lambda$ the values they have for air. The efficiency so calculated is called the *Air Standard Efficiency.*

## 93. **Tests.**

(i.) B.H.P. (see § 59).

(ii.) I.H.P. (see § 56).

(iii.) Heat supplied by combustion of fuel in given time.
= quantity of fuel used × lower calorific value.

(iv.) Heat carried in by jacket water = wt. of water used × its initial temperature.

(v.) Heat carried away by jacket water = wt. of water used × its temperature leaving jacket.

(vi.) Heat supplied to engine by air taken in.

Let $V$ = vol. of air used.

$v$ = specific volume.

$w$ = wt. of water vapour in the air.

$t$ = temperature of air, °C.

$$\text{Heat supplied by air} = \frac{V}{v} \,.\, k_p.\, t.$$

" " " water vapour $= wI_w$.

To find $w$: let $t^1$ = the dew point, °C. Find pressure of steam at temperature $t^1$ from tables; subtract this from the barometric pressure: this gives partial pressure of dry air ($p$, say). Then vol. of 1 lb. of dry air $= R(t+273)/p$. This will also be vol. of water vapour present at temperature $t^1$, and hence the weight of water vapour can be found.

(vii.) Heat carried away by exhaust gases.

= wt. of dry gases × $k_p$ × temperature °C.

+ wt. of steam × $I$ of steam and pressure and temp. of exhaust.

If an exhaust calorimeter is used: heat carried from engine in exhaust gases.

= heat carried away from calorimeter by gases + heat given to calorimeter cooling water.

## EXAMPLES.

**1.** A gas engine developing 10 I.H.P. consumes 180 ft.$^3$ of gas per hour, the calorific value of which is 320 Th. Units per ft.$^3$ The piston displacement is 1·2 ft.$^3$ and the clearance volume 0·24 ft.$^3$ Determine the thermal efficiency of the engine, and compare this with the efficiency of the standard engine. (Intercoll. Exam. Cambridge, 1911.)

$$\text{Heat supplied} = 180 \times 320 \text{ Th. Units per hour.}$$
$$= 960 \text{ Th. Units per minute.}$$

$$\text{The work done} = \frac{10 \times 33{,}000}{1400} = 236 \text{ Th. Units per min.}$$

$$\therefore \quad \text{thermal efficiency} = \frac{236}{960} = 0{\cdot}246, \text{ or } 24{\cdot}6\%.$$

Efficiency of standard engine

$$= 1 - \left(\frac{0{\cdot}24}{1{\cdot}44}\right)^{0{\cdot}4} = 0{\cdot}512, \text{ or } 51{\cdot}2\%.$$

**2.** A Diesel engine is supplied with oil whose calorific value is 8000 Th. Units per pound. On test it was found that the I.H.P. = 44·6, B.H.P. = 30·2, oil used per hour = 14·1 lbs., circulating water 870 lbs. per hour with a temperature rise of 40·5°C. Find the thermal efficiency, the mechanical efficiency, and estimate the heat lost in exhaust and radiation. (Mech. Sc. Trip., 1916.)

$$\text{Heat supplied} = \frac{14{\cdot}1 \times 8000}{60} \text{ Th. Units per minute.}$$
$$= 1880 \text{ Th. Units per minute.}$$

Indicated work

$$= \frac{44{\cdot}6 \times 33{,}000}{1400} = 1050 \text{ Th. Units per minute.}$$

Useful work

$$= \frac{30{\cdot}2 \times 33{,}000}{1400} = 710 \text{ Th. Units per minute.}$$

$$\text{Hence the thermal efficiency} = \frac{1050}{1880} = \underline{0{\cdot}56}$$

Hence the mechanical efficiency $= \frac{710}{1050} = \underline{0 \cdot 675}$

Heat carried away by cooling water per minute

$$= \frac{870}{60} \times 40 \cdot 5 = 588 \text{ Th. Units.}$$

We have then:

Th. Units per minute.

| | | | |
|---|---|---|---|
| Heat supplied | = 1880 | Useful work | = 710 |
| | | Mechanical losses = 1050 −710 | = 340 |
| | | Carried away by cooling water | = 588 |
| | | Exhaust and radiation | = 242 |
| | 1880 | | 1880 |

**3.** Describe how you would carry out a trial of a gas engine in order to obtain data for a heat balance sheet. In a trial of an engine rated at 50 B.H.P. the following data were obtained:

Duration of trial, 60 minutes.

Indicated horse-power, 55·9.

Brake horse-power, 48·1.

Total gas used at standard temperature and pressure, 880 cubic feet.

Lower calorific value of gas in T.U. per cubic foot, 310.

Jacket water per hour, 980 pounds.

Temperature of jacket water at entry, 7·2°C.

Temperature of jacket water at exit, 70°C.

Cooling water to exhaust calorimeter, 4300 pounds.

Temperature of water at entry to exhaust calorimeter, 6·6°C.

Temperature of water at outlet of exhaust calorimeter, 33·3°C.

Loss due to radiation in terms of gross heat supply, 5 %.

From these figures make out a balance sheet shewing the distribution of the heat supply to the engine. (Mech. Sc. Trip. 1910.)

Heat supplied by fuel $= 880 \times 310 = 273{,}000$ Th. Units.

Indicated work $= 55{\cdot}9 \times \dfrac{33{,}000}{1400} \times 60 = 79{,}000$ Th. Units.

Mechanical work

$$= 48{\cdot}1 \times \frac{33{,}000}{1400} \times 60 = 68{,}000 \text{ Th. Units.}$$

Heat carried in by cooling water $= 980 \times 7{\cdot}2 = 7050$

Heat carried away by cooling water

$$= 980 \times 70 = 68{,}500 \text{ Th. Units.}$$

Heat given up by exhaust gases

$$= 4300\,(33{\cdot}3 - 6{\cdot}6) = 115{,}000 \text{ Th. Units.}$$

Loss due to radiation

$$= 0{\cdot}05 \times 273{,}000 = 13{,}650 \text{ Th. Units.}$$

Hence we have:

Th. Units per hour, reckoned from O°C.

| | | | |
|---|---|---|---|
| Heat supplied by fuel = | 273,000 | Useful work | 68,000 |
| Heat in jacket water | 7,050 | Mechanical losses | 11,000 |
| | | Radiation loss | 13,650 |
| | | Heat carried away by jacket water | 68,500 |
| | | Heat from exhaust gases | 115,000 |
| | | Heat unaccounted for | 3,900 |
| | 280,050 | | 280,050 |

**4.** An engine working on the Otto cycle has a bore and stroke of 10″ and 15″; the compression ratio is 1 : 5 and the engine takes in mixture at 15°C., one cubic foot of the gas weighing 0·07 lb. The maximum temperature reached during explosion is 1800°C.; $\gamma = 1{\cdot}4$ and $k_v = 0{\cdot}18$. Find the indicated work per cycle. (Trin. Coll. Cambridge, 1914.)

Referring to Fig. 37:

$$T_1 = 288$$

$$T_2 = \left(\frac{v_1}{v_2}\right)^{0.4} T_1 = 5^{0.4} \times 288 = 548$$

$$T_3 = 2073$$

$$T_4 = \frac{2073}{5^{0.4}} = 1090.$$

Heat received

$= k_v (T_3 - T_2) = 0{\cdot}18 \times 1525 = 275$ Th. Units per lb.

Heat rejected

$= k_v (T_4 - T_1) = 0{\cdot}18 \times 802 = \underline{144}$ ,, ,,

$\therefore$ Work done $= \underline{131}$ ,, ,,

The stroke volume $= \dfrac{25\pi}{144} \times \dfrac{5}{4} = 0{\cdot}681$ ft.$^3$

This is the volume of gas sucked in per stroke.

$\therefore$ *wt* of gas used per stroke

$= {\cdot}681 \times 0{\cdot}07 = 0{\cdot}0477$ lbs.

Hence work done per stroke $= 131 \times 0{\cdot}0477 \times 1400$

$= 8750$ ft. lbs.

## MISCELLANEOUS EXAMPLES.

**1.** 18,000 gallons of water are being pumped per hour to a height of 70 ft. by a pump driven by an oil engine. The pump has an efficiency of 66%. Determine the I.H.P. of the oil engine if its mechanical efficiency is 75%. The kinetic energy of the water may be neglected.

If the oil is of sp. gr. 0·8, and the engine requires 0·1 gallon of oil per I.H.P. hour, what is the efficiency of the whole plant if the calorific value of the oil is 10,000 Th. Units per pound? (Intercoll. Exam. Cambridge, 1914.)

**2.** In the experiments of Osborne Reynolds to determine the mechanical equivalent of heat, a paddle was fixed to the shaft of an engine and rotated in water within a closed hollow vessel, freely mounted on the shaft, and prevented from turning round by weights attached to its side. At 300 R.P.M. of the engine 470 lbs. of water were found to have a rise of 99·5°C. in 62 minutes. Determine the mechanical equivalent of heat if the load on the side of the vessel was 141 lbs. at a leverage of 4 ft. (Intercoll. Exam. Cambridge, 1913.)

**3.** A bar of steel 3 sq. ins. section is quickly stretched in a testing machine by 1/2000 of its length. What change would be required in the balancing weight as the temperature recovers its original value if the stretch is maintained constant? coeff. of expansion 0·000013/°C.; sp. ht. = 0·1098; sp. gr. = 7·55. E. = $30 \cdot 10^6$ lbs./in.$^2$) (R.N.C. Greenwich, 1912.)

**4.** The thrust bearing of a liner is water cooled. The water used is 5750 lbs. per hour, and the temperature rise is 9°C. What horse-power is wasted?

**5.** In the brake-horse-power test of a certain gas engine the nett load on the brake is 21 lbs., the diameter of the wheel is 5′6″, and the speed 130 R.P.M. How much heat is generated per minute?

**6.** In the trial of a gas engine the following observations were made:

Duration of trial 60 minutes, I.H.P. 60, B.H.P. 52, total gas used 950 cubic ft., total air used 8000 cubic ft. both being at standard temperature and pressure; higher calorific value of the gas 360 Th. Units per standard cubic foot, jacket water per hour 1060 lbs., temperature of jacket water at entry 12°C., at exit 75° C., cooling water to exhaust calorimeter 5000 lbs., temperature of water at entry to exhaust calorimeter 12°C., at exit 36°C., temperature of gas in the exhaust pipe 50°C. The specific heat of the gases is 0·24 and the temperature of the gas and air supply is 15°C. Make out a complete balance-sheet shewing the distribution of the heat supply to the engine. (Intercoll. Exam. Cambridge, 1913).

**7.** The following data were obtained in a test of a Diesel engine:

Duration of test, 60 minutes.

I.H.P. = 12·95.; B.H.P. = 9·94.

Oil used 5·44 lbs.; lower calorific value 9340 lb. cal. per lb.

Cooling water used 700 lbs.; rise of temperature, 16·2°C.

Calculate the thermal efficiency relative to I.H.P. and to B.H.P., and find what percentage of the heat supplied is not accounted for in the above observations. What becomes of the heat which is not accounted for?

The air compressor is driven direct from the engine shaft and takes 1·1 H.P., of which 75% may be taken as usefully returned to the engine cylinder in the air supply. If this is taken into account how does it affect the values of the thermal efficiency obtained above? (Mech. Sc. Trip., 1914.)

**8.** A gas engine has a stroke volume of 1450 ins.$^3$ and a clearance volume of 250 ins.$^3$, and uses 15 ft$^3$. of gas per I.H.P. hour, the calorific value of the gas being 310 Th. Units

per ft.$^3$ Find the thermal efficiency of the engine and compare this with the efficiency of the standard engine. (Intercoll. Exam. Cambridge, 1914.)

**9.** In a trial of a Diesel oil engine the oil used had a calorific value of 10,700 thermal units per pound, and 18·5 lbs. of oil were used per hour. The B.H.P. was 40·4 and the I.H.P. 52·6. Cooling water passed through the jackets at the rate of 30·2 lbs. per minute and its rise of temperature was 29°C. It was calculated that the exhaust gases carried off 940 thermal units per minute. Determine the mechanical efficiency, the heat used per I.H.P. minute, and draw up a heat account, calculating the percentage of heat unaccounted for. (Intercoll. Exam. Cambridge, 1908).

**10.** A balloon of 5000 cubic ft. capacity is to be so far filled with hydrogen at 30″ of mercury and 15°C., that after ascending to a height where the pressure is 20″ of mercury and the temperature 0°C., the silk envelope is then fully extended, no gas having escaped. Calculate the mass of hydrogen required and its original volume. The density of hydrogen = 0·0056 lbs. per ft.$^3$ at standard temperature and pressure. (Mech. Sc. Trip., 1912.)

**11.** A cubic ft. of air weighing $\frac{1}{4}$ lb. expands from a pressure of 100 lbs./in.$^2$ to a volume of 10 ft.$^3$ at 20 lbs./in.$^2$, the pressure falling uniformly. Find the heat taken in by the air. (Mech. Sc. Trip., 1916.)

**12.** One pound of air at 0°C. is expanded at constant pressure to three times its initial volume. Calculate the final temperature, the work done, and the heat supplied.

**13.** One pound of water at $T_1°$ abs. is mixed with one pound of water at $T_2°$ abs. Show that there is a gain of entropy equal to

$$\log_e \frac{(T_1+T_2)^2}{T_1\,T_2}.$$

(Intercoll. Exam. Cambridge, 1912.)

**14.** Prove that if ever a method is found for reversing the conduction of heat, a perpetual motion with enormous power behind it will have been discovered. (Mech. Sc. Trip., 1912.)

**15.** One pound of air, initially at 10°C., 15 lbs./in.$^2$ is heated at constant volume to a temperature of 350°C.; it is then expanded adiabatically and brought back to its original condition by isothermal compression. Draw the T-$\phi$ diagram and calculate the efficiency of the cycle.

**16.** Ten cubic feet of gas at 100 lbs./in.$^2$, 150°C. are expanded adiabatically to a pressure of 20 lbs./in.$^2$, and then compressed to the original volume according to the law $pv^{1 \cdot 1}$ = const., and finally heated at constant volume to the original pressure. Determine the work done, and the efficiency of the cycle. (Intercoll. Exam. Cambridge, 1913.)

**17.** A deflated gas bag of small capacity is charged from a steel bottle containing air under high pressure, and at 10°C. When the connection between the bag and the bottle is shut, the pressure in the bag is 15·5 lbs/in.$^2$ abs. The barometer stands at 29·2″, there is no loss of heat and no work done in stretching the bag. Find the temperature of the air in the bag.

**18.** A quantity of gas of volume $V$ and at atmospheric pressure $\Pi$ is confined behind a piston of area $A$. A spring bears on the other side of the piston and is compressed when the piston moves out. If the force necessary to compress the spring a unit distance be $\lambda$, show that the quantity of heat, in work units, that must be given to the gas so that it slowly expands and compresses the spring a distance $x$ is:

$$\frac{\gamma+1}{\gamma-1} \cdot \frac{\lambda x^2}{2} + \frac{x}{\gamma-1}\left(\gamma \Pi A + \frac{\lambda V}{A}\right).$$

(Intercoll. Exam. Cambridge, 1912.)

**19.** Steam initially at 200°C., and dryness 0·8 expands adiabatically to a temperature of 90°C. Find the dryness fraction at the end of expansion. If the expansion curve is to be represented by $pv^n$ = constant., find $n$.

**20.** Steam at 210 lbs./in.$^2$ abs. and dryness 0·8 is throttled down to a pressure of 40 lbs/in.$^2$ abs. Find the final state of the steam.

**21.** How much heat must be given to 10 lbs. of water at 10°C. in order to convert it into steam at 220 lbs. per sq. in., with 100°C. of superheat.

**22.** On a temperature-entropy diagram for steam draw the constant volume line from 60 lbs./in.$^2$ to 20 lbs./in.$^2$, taking the volume as that of 1 lb. of dry steam at 60 lbs./in.$^2$

**23.** A boiler of 200 cubic feet capacity contains equal volumes of water and steam when the pressure is 30 lbs./in.$^2$ Find the quantity of heat which must be given to the boiler to raise the pressure to 120 lbs/in.$^2$ abs. If the efficiency of the boiler is 69%, and the calorific value of the coal 8500 Th. Units per lb., how much coal will be consumed? (Intercoll. Exam. Cambridge, 1911.)

**24.** The average total pressure of a slide valve on its seat is 2 tons, and the coefficient of friction is 0·08. The travel of the valve is 3·5″, and the speed of the engine 200 R.P.M. If half the heat developed by friction goes to evaporate water in the steam passing into the cylinder, find the water evaporated per lb. of steam when there are 42 lbs. of steam used per minute, the steam chest pressure being 150 lbs./in.$^2$ (Intercoll. Exam. Cambridge, 1913.)

**25.** 10 lbs. of water at 15°C. are introduced into an exhausted vessel of 20 cubic feet capacity. How much water will be evaporated, and by how much will the temperature fall? The vessel is then heated until the pressure rises to 100 lbs. per sq. in. absolute: how much heat is taken in, and what is then the internal energy?

Steam is then blown off until the pressure falls to 50 lbs. per sq. in. absolute: how much steam is blown off? (Trin. Coll. Cambridge, 1914.)

**26.** The expansion curve of an indicator diagram is found to have the equation $pv^{1\cdot1} = \text{const.}$; determine how much heat has been taken in from the cylinder walls, per pound of steam, as the steam expands from the pressure 100 lbs./in.$^2$ abs. to 20 lbs./in.$^2$ abs., the steam being dry at the higher pressure. (Mech. Sc. Trip., 1912.)

**27.** If the dryness fraction of a vapour remain constant during adiabatic expansion, show that the relation between the latent heat and the temperature must be of the form

$$T = ae^{-\frac{qL}{\sigma T}}$$

where $a$ is a constant, and $\sigma$ the specific heat of the liquid, (Mech. Sc. Trip., 1913.)

**28.** A steam main 6 inches in external diameter conveys 120 lbs. of steam per minute from a boiler working at 200 lbs. pressure per square inch absolute to an engine 90 feet away. The pressure at the engine stop valve is 150 lbs. absolute and the dryness fraction of the steam is 0·93. Calculate the loss per minute in T.U. per square foot of pipe due to conduction and radiation, if the effective length of the pipe is 100 feet The pipe is afterwards covered by lagging and the steam then reaches the engine at a pressure of 190 lbs. per square inch absolute and dryness fraction 0·97. Calculate the saving in T.U. per minute effected by the lagging. (Mech. Sc. Trip. 1910.)

**29.** A boiler contains 3000 lbs. of steam and water at 120 lbs./in.$^2$ abs., half the total volume being water. Find the amount of steam which must be blown off to reduce the pressure to 50 lbs./in.$^2$ abs.

**30.** The indicator diagram shewn (Fig. 39) is taken from a single-acting engine using 1200 lbs. of steam per hour and

making 90 revolutions per minute. The clearance volume is 10 per cent. of the stroke volume. The stroke of the engine is 2 feet and the diameter of the cylinder is 18 inches. Determine the I.H.P. of the engine and find the dryness of the steam at the point marked $A$ in the diagram. Atmospheric pressure = 15 lbs. per sq. in. (Intercoll. Exam. Cambridge, 1913.)

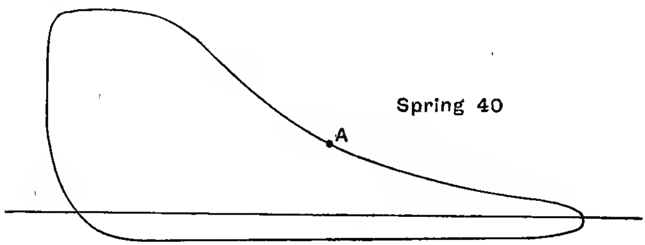



Fig. 39.

**31.** Two points on the expansion line of an indicator card gave

$$p_1 = 75 \text{ lbs. per sq. in.} \quad \text{and} \quad p_2 = 15 \text{ lbs. per sq. in.}$$
$$v_1 = 1{\cdot}3 \text{ cub. feet} \qquad\qquad v_2 = 5{\cdot}1 \text{ cub. feet.}$$

the pressures being measured from the atmospheric line and the volumes from the beginning of the stroke. Atmospheric pressure 15 lbs. per sq. in.

The total mass of steam expanding in the cylinder was estimated at 0·48 lbs. and the volume of the clearance space 0·46 cub. feet. Assuming the expansion line to be represented by the equation $PV^n$ = constant, find the work done by the steam between the points (1) and (2) and the interchange of heat between the steam and the cylinder. (Intercoll. Exam. Cambridge, 1914.)

**32.** The indicator diagram taken from the end of a single acting cylinder is as shewn in Fig. 40. The cylinder is 1 foot 3 inches in diameter and the stroke is 1 foot 9 inches. Determine the indicated horse-power of the engine.

The clearance volume at either end of the cylinder is 10 per cent. of the piston displacement and the air pump discharge is 40 lbs. per min. If the atmospheric pressure is 14·7 lbs. per sq. inch, determine the dryness of the steam at the points $A$ and $B$ and shew on the diagram supplied to you the points on the saturation curve corresponding to $A$ and $B$. (Mech. Sc. Trip., 1912.)

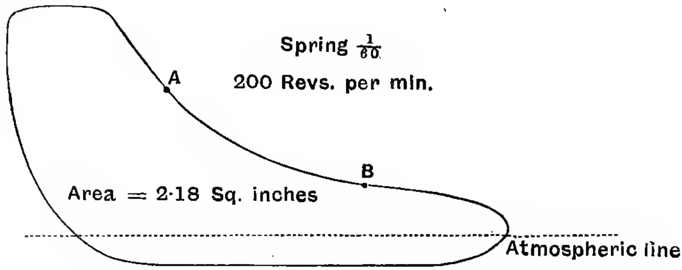



Fig. 40.

**33.** A steam engine consumes 3000 lbs. of steam per hour and develops 320 I.H.P. The steam is supplied at 150 lbs./in.$^2$ abs. and 400°C. The pressure in the condenser is 1·25 lbs./in.$^2$ abs. What is the thermal efficiency, and what would it have been if the engine had followed the Rankine cycle?

**34.** A boiler supplies steam at a pressure of 200 lbs. per sq. in. abs., the dryness fraction being 0·9. After passing through a reducing valve, with a reduction of pressure to 160 lbs. per sq. in. the steam enters the high-pressure cylinder of a compound engine where it expands adiabatically to 40 lbs. per sq. in. It is exhausted from this cylinder at 22 lbs. per sq. in. entering the low-pressure cylinder, where it expands adiabatically to 8 lbs. per sq. in., exhaust taking place at 2 lbs. per sq. in. Estimate the volume of the steam per pound at cut-off in each cylinder and at the beginning of exhaust in the low-pressure cylinder. (Mech. Sc. Trip., 1910.)

**35.** Dry steam is supplied to a single-cylinder steam engine at 102 lbs. per sq. in. absolute pressure and the engine

uses 19 lbs. per I.H.P. per hour. The steam-jacket condenses 0·95 lb. of steam per I.H.P. per hour at the temperature of supply. The engine has a jet condenser to which water is supplied at 10°C. Neglecting loss of heat by radiation, find how many pounds of water must be supplied to the condenser per pound of steam supplied to the engine, in order that the air pump discharge may have a temperature of 35°C. (Mech. Sc. Trip., 1906.)

**36.** Dry steam is admitted to an engine cylinder at a pressure of a 100 lbs. per sq. in. and 20% of it is condensed during admission, without fall of pressure. The steam expands down to a pressure of 20 lbs. per sq. in. and during expansion one half of the heat absorbed by the walls is returned to the steam at a uniform rate as the temperature falls. Find the dryness fraction of the steam at the end of expansion. Find also the amount of work obtainable in the cylinder per pound of steam from the returned heat.

Shew that if the steam expand adiabatically to the same final volume the final pressure will be a little below 18 lbs. per sq. in. (Mech. Sc. Trip., 1914.)

**37.** Calculate the work available per pound of steam in a Rankine Cycle, the exhaust being at 2 lbs./in.$^2$ abs : (*a*) when the supply pressure is 150 lbs./in.$^2$ abs. and the steam is dry and saturated; (*b*) when dry steam at 150 lbs./in.$^2$ abs. is throttled just before admission to 100 lbs./in.$^2$ abs. (R.N.C. Greenwich, 1912.)

**38.** The temperature inside a surface condenser is 46·6°C. and the pressure 2 lbs. per sq. in. The steam space is of 40 cubic feet capacity. Shew that the weight of air present is 0·093 lb., and is 53 per cent. of the weight of steam present.

The engine with this condenser is of 50 I.H.P. and consumes 16 lbs. of steam per I.H.P. hour. Assuming that the exhaust steam entering the condenser is free of air, determine the proper volume of the air pump cylinder, making 120

strokes per minute necessary to overcome a leakage of air into the condenser at the rate of 1 lb. per hour. (Intercoll. Exam. Cambridge, 1914.)

**39**. Dry hot air is passed into a cooling tower containing tiles which are kept wetted by a supply of water at 15°C. The air emerges saturated with water-vapour at a temperature of 60°C. Shew that the quantity of water evaporated is about 0·15 pound per pound of air, and that the temperature of the entering air is about 440°C. The density of dry air may be taken as ·08 pound per standard cubic feet. (Mech. Sc. Trip., 1914.)

**40**. A fluid expands adiabatically from 120 lbs. absolute to 15 lbs. absolute in an expanding nozzle according to the law $pv^{1\cdot13}$ = constant. If the flow per second is 0·2 lb., find the area of the minimum section and the area of the outlet section of the nozzle. The specific volume at the higher pressure is 3·7 cubic feet per pound. (Mech. Sc. Trip., 1910.)

**41**. Steam is discharged through a nozzle from an initial pressure of 160 lbs/in.$^2$ abs. into the casing of a De Laval turbine where the pressure is 2·5 lbs./in.$^2$ abs. Neglecting osses, find the velocity of discharge.

**42**. Steam initially dry and at rest under 180 lbs. pressure flows down a nozzle with no external communication of heat. It is estimated that at the section where the pressure is 40 lbs. per. sq. in. the kinetic energy of the steam is 20% less than it would be at that pressure in the case of frictionless flow. Find the percentage increase of section, at this pressure, as compared with frictionless flow, in order that the discharge may be the same in both cases.

Also if the $pv$ curve during the expansion down the nozzle is of the form $pv^n$ = a constant, find the value of $n$, and determine how much heat has been communicated, to each pound of steam, between the given pressures. (Mech. Sc. Trip., 1910, B.)

**43.** An ejector is supplied with dry steam at 245 lbs/in.$^2$ abs., and lifts water from a tank at the rate of 48 tons per hour, the lift being 12 feet. The water in the tank is at 5°C., and the discharged water at 28°C. Find the steam used per ton of water, and the efficiency of the ejector considered as a pump.

**44.** In a Curtis turbine, with an initial pressure of 250 lbs./in.$^2$ abs. and final pressure 1 lb./in.$^2$ abs. the steam is expanded in 7 stages, in which the first develops $\frac{1}{4}$ of the total power. If 20% of the available energy is lost in friction through the turbine, find the number of heat units converted into kinetic energy in the first stage. If 14% of the energy is lost in the nozzles, what is the velocity of exit from the nozzles of the first stage ? (R.N.C. Greenwich, 1910.)

**45.** The blades of a De Laval turbine have angles of 35° at inlet and exit, and the nozzle makes an angle of 18° with the plane of rotation. The steam velocity at exit from the nozzle is 3600 ft./sec. When the speed of the vanes is 1200 ft./sec., will the steam impinge on the blades without shock ? Assuming that the relative velocity at exit is 80% of the relative velocity at inlet, calculate, at the above speed of the vanes, the work given to the vanes per pound of steam and the internal efficiency of the turbine. (R.N.C. Greenwich, 1909.)

**46.** One stage of an impulse turbine of the Curtis type has three moving and two fixed rings. The blades on the moving rings have the angles of exit and of entry equal to one another, and these are the same in all three rings. Steam issues from the nozzles with a velocity of 2650 ft./sec., and the nozzles make an angle of 20° with the direction of motion of the blades. The moving blades have a velocity of 530 ft./sec. Find the angles of entry and exit for both rings of fixed blades, and the common angle of exit and entry for the moving blades.

Find also the horse-power per pound of steam passing per second. All frictional effects are to be neglected (Mech. Sc. Trip., 1914, B.)

**47.** Steam issues from the nozzles of a Laval turbine at a velocity of 3325 ft./sec. The axes of the nozzles are severally placed at angles of 18 degrees with the plane of revolution of the wheel. Find by measurement from a diagram of velocities, drawn for maximum efficiency,

(1) the angle of the vane at entry,

(2) the absolute velocity of the steam at discharge from the wheel, assuming that the turbine blades are symmetrical,

(3) the efficiency of the turbine,

(4) the horse-power of the turbine, assuming that 900 pounds of steam are used per hour.

All frictional and eddy losses are to be neglected. (Mech. Sc. Trip., 1907.)

**48.** In an ammonia compression refrigerator the temperature in the evaporator is $-10°$C., and at the end of compression the pressure is 125 lbs./in.$^2$ The liquid passes to the expansion valve at 20°C. If the compression be adiabatic, what will be the coefficient of performance if the vapour is just dry (*a*) at the end, (*b*) at the beginning, of compression? (R.N.C. Greenwich, 1909.)

**49.** An ammonia refrigerating machine is required to produce 3 tons of ice per hour from water at 12°C. The temperatures of evaporation and condensation of the ammonia are $-10°$C. and 25°C. respectively, and the condensed ammonia is driven through an expansion valve without preliminary cooling. Find the horse-power required in the compressor if the vapour is compressed adiabatically and is just dry at the end of compression. The latent heat of water may be taken as 80.

If the vapour is just dry at the beginning of compression and is compressed adiabatically to the same final pressure, find the amount of superheat at the end of compression and the horse-power required. (Mech. Sc. Trip., 1914.)

**50.** Figure 41 shows a sketch of a portion of Mollier's $\phi I$ diagram for carbonic acid. Explain the construction of the diagram and sketch on the figure one or two lines of constant pressure in the region of superheat.

Describe the use of this diagram in refrigeration problems and make an estimate from it of the coefficient of performance of a $CO_2$ cycle working between 20°C. and − 20°C., when the vapour is dry at the end of the compression. Check your estimate by making an evaluation of the coefficient from the data given in the $CO_2$ table. (Mech. Sc. Trip., 1915, B.)

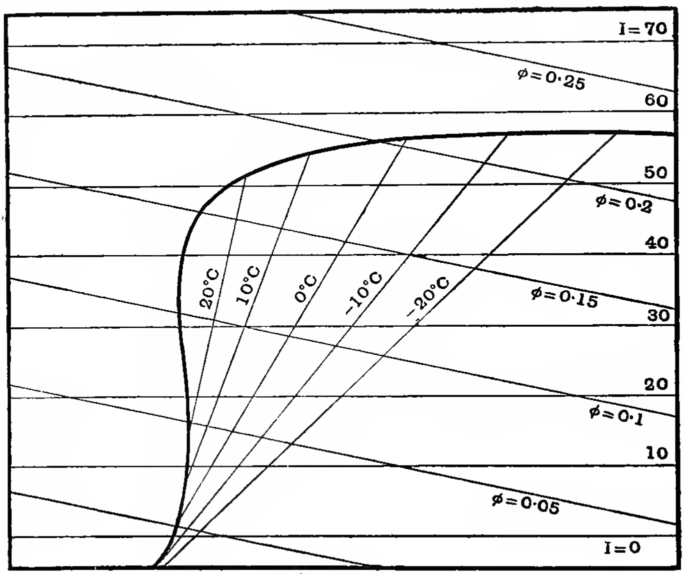



Fig. 41.

**51.** Two refrigerating machines, one using $SO_2$ and the other $NH_3$, work between the same limits of temperature,

viz. 10°C. and $-$ 34·4°C. In each case the vapour is dry and saturated at the end of compression, and expansion takes place through a throttling valve. Determine in each case the refrigerating effect per pound of substance and the coefficient of performance. Also show that, if the machines are to have the same refrigerating effect per stroke, the volumes of their compressing cylinders must be approximately in the ratio 11 : 4. (Mech. Sc. Trip., 1908.)

**52.** A small reservoir for compressed air has a volume of 2 cubic feet, and is being charged from the atmosphere by means of a single-acting pump of diameter 6 inches and stroke 9 inches. At the beginning of one cycle of the pump, the reservoir pressure is 20 lbs. per sq. inch by gauge. Find the pressure after one cycle of the pump, neglecting clearance and assuming that the air remains at the temperature of the atmosphere, 15°C. The atmospheric pressure is 14·7 lbs. per sq. inch.

Find also the work done by the pump and the mass of air pushed into the reservoir. (Intercoll. Exam. Cambridge, 1911.)

**53.** A single-acting reciprocating air engine, with a stroke of 6 inches and piston diameter 4 inches, is being driven from a reservior of compressed air of 10 cubic feet capacity; the pressure of the reservoir falling as the air is used in the engine. At the beginning of a particular cycle of the engine, the reservoir pressure is 100 lbs. per sq. inch by gauge and the temperature 15°C. Find the work done in this cycle if the engine cuts off at $\frac{1}{4}$ stroke and exhausts into the atmosphere. Neglect clearance and assume the process adiabatic.

Find also what will be the temperature of the reservoir after the cycle.

Pressure of atmosphere 14·7 lbs. per sq. inch. (Intercoll. Exam. Cambridge, 1912.)

**54.** The piston of an air compressor displaces 8 cubic feet per stroke and makes 120 strokes per minute. It takes in atmospheric air at 15°C. and compresses it, according to the law $PV^{1.25}$ = constant, up to 75 lbs. gauge pressure; finally delivering it at this pressure into a reservoir. Assuming no slip past the valves, no loss of head through them, and in the first place no clearance, calculate the work done upon the air in foot-pounds per minute, and the temperature at which it enters the reservoir. In the second place, if the clearance were 10 per cent. of the piston displacement, how would the work done per minute and the volumetric efficiency of the compressor be affected? Atmospheric pressure = 14·7 lbs. per sq. in. (Mech. Sc. Trip., 1906.)

**55.** The mean pressure during the suction stroke of a 4-cycle gas engine is 2 lbs./in.$^2$ below atmosphere. Shew that the temperature of the charge will be raised about $11\frac{1}{2}$°C. by the work done on it in drawing it in, the temperature of the outside air being 17°C. (Mech. Sc. Trip. B., 1910.)

**56.** A three-stage compressor, which was designed to give maximum efficiency when pumping against a constant back pressure $p_3$ from the initial pressure $p_0$, the interstage pressures being $p_1$ and $p_2$, is used to charge an air-vessel of volume $V$ from atmospheric pressure $p_0$ up to the pressure $p_3$. Calculate the work done by the first-stage cylinder while the pressure in the air-vessel rises to $p_1$; also the work done in the first stage cylinder while the pressure in the air-vessel rises from $p_1$ to $p_2$. (R.N.C., Greenwich, 1912.)

**57.** A gas-engine uses producer gas of the following composition by volume

| | Per Cent. |
|---|---|
| CO | 25 |
| H | 14 |
| $CO_2$ | 6 |
| N | 55 |
| | 100·0 |

The exhaust-gases contain 15 per cent. of $CO_2$. Shew that the ratio of the volumes of air to gas taken into the engine is 1·41. Air contains 21 per cent. of its volume of oxygen.

What percentage of oxygen would you expect to find in the exhaust-gases? (Mech. Sc. Trip., 1914.)

**58.** In the case of a land boiler fitted with a feed heater in the flue, the coal burnt per hour was 125·5 lbs.; the feed per minute was 22·5 lbs., the temperature of the products of combustion in the smoke-box and chimney bottom were 353°C. and 198°C. respectively, and the temperature of the feed water entering and leaving the feed heater were 64°C. and 109°C. respectively. If the specific heat of the products of combustion is 0·24 and the temperature of the boiler steam is 193°C., calculate the lbs. of air supplied per lb. of coal burnt; and also (assuming the boiler steam dry) calculate the evaporation per lb. of coal from and at 100°C. (R.N.C., Greenwich, 1908.)

**59.** In the published results of a boiler test, the fuel and flue gas analyses are given as follows:

| | Coal Analysis by Weight | Flue Gas Analysis by Volume | |
|---|---|---|---|
| Carbon | 83·44% | $CO_2$ | 11·7% |
| H | 3·99 | O | 3·85 |
| O | 2·88 | CO | 0·45 |
| Ash | 9·69 | N | 84·0 |

Are these figures consistent with reasonably complete combustion of the coal, and how could they be explained? The atomic weight of carbon is 12, that of oxygen is 16, and the ratio of the nitrogen to oxygen in the air is 3·76 by volume. (Mech. Sc. Trip., 1913.)

**60.** In the trial of a gas engine, the volume analysis of the gas supplied to the engine was $CH_4$, 33%; $C_2H_4$, 5%; H, 42%; CO, 7%; N, 10%; $CO_2$, 3%. The volume analysis of the exhaust gases was $CO_2$, 5·8%; O, 10·4%; N, 83·8%. Calculate the number of cubic feet of air

necessary for the complete combustion of one cubic foot of gas, and the excess air supplied to the engine per cubic foot of gas. Take the composition of air by volume as $O_2$ 21%; N, 79%. (R.N.C. Greenwich, 1911.)

**61.** A gas engine working the Otto cycle has a cylinder 10 inches diameter and a stroke of 20 inches. The compression space is $\frac{1}{5}$ of the stroke volume. The inlet valve closes at the out-centre and the cylinder contents then consist of air having a temperature of 40°C. and a pressure of 14·7 lbs. per square inch. The pressure reached in compression is 160 lbs. per square inch absolute, the compression line of the indicator diagram following the curve $pv^{1 \cdot 33} = \text{const.}$

Find :—

(1) The mean temperature of the air at the end of compression.

(2) The temperature reached at a point in the interior of the cylinder where there has been no loss of heat.

(3) The quantity of heat lost to the cylinder walls in the course of compression. (Mech. Sc. Trip., 1907.)

**62.** The following are the results of the trial of a gas-engine working the Otto cycle, and governing by hit and miss :—

Cylinder diameter, 10 inches.

Stoke, 18 inches.

Gas used at full load, ·087 cub. ft. per explosion.

The engine misses once in 10 cycles.

Calorific value of gas, 308 C.T.U. per cubic foot at the temperature and pressure at which it is used.

Brake Horse-power, 25.

When the brakes are taken off the engine fires once in six cycles, the mean pressure of the explosion diagrams being

90 pounds per square inch. The speed is 180 revs. per minute whether loaded or unloaded.

From these data estimate the indicated horse-power of the engine, when giving the load of 25 B.H.P., and the thermal efficiency (on the indicated horse-power.) (Mech. Sc. Trip., 1907.)

**63.** A gas engine works on an ideal cycle with adiabatic compression and expansion, receiving and rejecting heat only at constant volume. Obtain the expression for its efficiency. In such an engine the piston displacement per stroke is 1 cubic foot, the clearance volume 0·2 cubic foot, and at the beginning of compression the temperature of the cylinder contents is 333°C. abs., pressure being atmospheric. The engine receives 0·06 cubic foot of gas per cycle (calorific value 330C. Th. Units per cubic foot). Atmospheric pressure = 14·7 lbs. per sq. in.

Find :—

(*a*) Weight of cylinder contents.

(*b*) Pressure and temperature at end of compression (take $\gamma = 1{\cdot}38$.)

(*c*) Rise of temperature during explosion (neglect jacket loss and take $k_v = 0{\cdot}18$).

(*d*) Pressure at end of explosion.

(*e*) Temperature and pressure at end of expansion.

(*f*) Efficiency of the cycle.

(*g*) Efficiency of an engine working on a Carnot cycle between the same highest and lowest temperatures. (Mech. Sc. Trip., 1906.)

**64.** The following data were obtained in a complete test of an oil-engine :—

Speed, 300. Barometer, 29·4 inches. B.H.P. 4½.

Calorific value of oil used, 12,000 Th. Units per lb.

Oil entering engine per min., 0·7 lb.

Air ,, ,, ,, 1·92 ,,

Atmospheric temperature, 17° C. Dew point, 12°C.

Jacket water: Quantity, 15 lb. per min. Temperature: Inlet, 17°. Outlet, 37°.

Temperature of exhaust gases, 520°C.

The chemical formula for petroleum may be taken as $C_{14}H_{30}$.

Draw up a complete "heat balance sheet" for the engine, and deduce its thermal efficiency. (T.T., 1912.)

# INDEX

*The numbers refer to pages.*

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